A DISCRETE-ELEMENT ANALYSIS FOR LARGE DEFLECTION OF THIN PLATES: A COMBINED EFFECT OF GEOMETRIC AND MATERIAL NONLINEARITIES

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LIST OF SYMBOLS

a	bar in the x direction
$a_{i,j}$	stiffness matrix coefficient
A_a	area of the elastic bar in the x direction
A_b	area of the elastic bar in the y direction
A_c	area of the elastic bar in the diagonal direction
b	bar in the y direction
$b_{i,j}$	stiffness matrix coefficient
c	bar in the diagonal direction
CM^x, CM^y	model couple moment
$c_{i,j}$	stiffness matrix coefficient
CF	couple force
$d_{i,j}$	stiffness matrix coefficient
D_x, D_y	plate bending stiffness in the x and y directions
D_{xy}	plate twisting stiffness
$e_{i,j}$	stiffness matrix coefficient
E_x, E_y	modulus of elasticity
E_{xm}, E_{ym}	equivalent elastic modulus for the membrane model
E_{xb}, E_{yb}	equivalent elastic modulus for the bending model
$f_{i,j}$	stiffness matrix coefficient
$\{F_T\}$	vector of total load
$\{F^\ell\}$	load vector for ℓth loading step
$F_{i,j}$	load applied to node i,j of the composite model

$(f_B)_{i,j}$	load carried by the bending model at node i,j
$g_{i,j}$	stiffness matrix coefficient
G_{xy}	shear modulus
h_x, h_y, h_z	discrete element dimension
i,j	node point identifications
$[K_B]$	stiffness matrix of the bending model
$[K_M]$	geometric stiffness matrix of the membrane model
M_x, M_y	continuum bending moment
M_{xy}, M_{yx}	continuum twisting moment
M^x, M^y	model bending moment
M^{xy}, M^{yx}	model twisting moment
$\bar{M}_x$, $\bar{M}_y$, $\bar{M}_{xy}$, $\bar{M}_{yx}$	resultant moment
N_x, N_y	continuum membrane force (axial)
N_{xy}, N_{yx}	continuum membrane force (shear)
$\bar{N}_x$, $\bar{N}_y$, $\bar{N}_{xy}$, $\bar{N}_{yx}$	resultant force
$P_{i,j}$	stiffness matrix coefficient
p^x, p^y, p^{c1}, p^{c2}	bar force in the membrane model
q	uniformly distributed lateral load
$q_{i,j}$	stiffness matrix coefficient
$r_{i,j}$	stiffness matrix coefficient
$R_{i,j}$	transverse resistance of the membrane model at node i,j
$\{R\}$	vector of the transverse resistance of the membrane model
R^x, R^y	model rotational stiffness
$s_{i,j}$	stiffness matrix coefficient
$S_{i,j}$	transverse support spring stiffness

S_x, S_y, S_z	stress deviation
t	thickness of the plate
$t_{i,j}$	stiffness matrix coefficient
T^x, T^y	model twisting couple
u, v	in-plane displacement
$u_{i,j}$	stiffness matrix coefficient
V	shear force
w	transverse displacement
$\{w\}$	transverse deflection vector
ϕ_x, ϕ_y, ϕ_{xy}	plate curvature and twist
ε_x, ε_y, ε_{xy}	lamina strain
ε_{xm}, ε_{ym}, ε_{xym}	midsurface strain
ν_{xm}, ν_{ym}	equivalent elastic Poisson's ratios for membrane model in the x and y directions
ν_{xb}, ν_{yb}	equivalent elastic effect of the x and y curvatures on those of y and x directions
σ_x, σ_y, τ_{xy}	normal and shearing stresses
$\Delta\varepsilon_x^p$, $\Delta\varepsilon_y^p$, $\Delta\varepsilon_{xy}^p$	increments of normal and shearing plastic strain
$\Delta\varepsilon^p$	increment of equivalent plastic strain
σ_e	effective stress
ε_e	effective strain
$d\lambda$	constant
α	constant

CHAPTER I

INTRODUCTION

The plate is one of the most common structural elements used in construction, ship building, and aerospace industries. The formulation of the relationship between the lateral loads on the plate and the resulting plate deformations is a function of the deformation and the material behavior during the course of loading. The analysis of a plate subjected to lateral loads may be carried out by means of the linear elastic theory, elastic large deformation theory, or elastic-plastic large deformation theory, depending on the maximum deflection to thickness ratio and the material behavior.

The linear elastic theory is developed on the assumption that the deformations are linearly related to loads and are small compared to the thickness of the plate.

The elastic large deformation theory explores the resistance of the plate to lateral loads up to the elastic limit of the material. In this theory, the membrane forces produced as a result of the strains in the middle surface provide resistance for lateral loading.

The elastic-plastic large deformation theory not only takes into consideration the reserve strength of the plate due to membrane action, but also accounts for changes in the material property that occur during loading. Equations that relate the lateral loads to large deformation of plates are not amenable to closed-form solution.

1.1 Background

Leesavan (1) proposes a numerical solution for geometrically nonlinear problems of large deformation of thin plates in the elastic range. In his work the behavior of the plate is simulated by a discrete-element model made of two components: bending and membrane components. The bending properties of the plate are assumed to be lumped at the node points of a bending model. The stretching is assumed to be resisted by a membrane model consisting of a truss structure. In his solution deformations are determined by applying the loads to the nodes of the model and achieving the conditions of equilibrium and compatibility of displacements at all grid points of the model components through a repetitive solution scheme.

1.2 Objective and Scope

The objective of this work is to present a numerical method for analyzing the large deflection of thin plates in the elastic-plastic range--a problem that invovles the combined effects of geometric and material nonlinearities. This work is an extension of Leesavan's (1) to include material nonlinearity. The computer program presented in this study can be used to evaluate the deformations of plates made of elastic, elastic-perfectly plastic, or work-hardening materials subjected to various boundary conditions.

1.3 Method of Analysis

In the present study the physical model developed by Leesavan (1) is employed for analyzing the large deflection of plates with material nonlinear behavior. The solution procedure consists of four steps:

1. Portions of the total load are applied either to the bending or the composite model.

2. At any stage of the gradual loading of the model, equivalent elastic material properties are calculated for the node points of the discrete-element model.

3. The nonlinear problem at any stage of loading is converted into an equivalent linear problem through the use of equivalent elastic constants.

4. Each of the linear problems is solved by means of an iterative scheme that utilizes a secant stiffness. This secant stiffness contains the effect of either material or material and geometric nonlinearities.

To demonstrate the validity and accuracy of the proposed method of solution, the results of analysis of several problems are included and compared with the analytical and experimental works of other authors. These results are in excellent agreement with the existing analytical solutions and experimental data reported by other researchers.

CHAPTER II

REVIEW OF THEORY AND LITERATURE

This chapter presents a brief account of the equilibrium formulation of thin plates and the principles and equations of the theory of plasticity. The numerical and experimental works on elastic and elastic-plastic large deflection of plates are also reviewed.

2.1 Theory of Plates (General)

With regard to the ratio of their dimensions, plates are classified as thick or thin. A thin plate has a thickness that is small in comparison to its other dimensions. In general, a three-dimensional analysis approach is required for calculating deflections and stresses in any plate subjected to lateral loads. For a thin plate, however, a sufficiently accurate two-dimensional analysis method may be used by making certain simplifying assumptions (2).

Depending on the material behavior and the ratio of plate deflection to thickness, the problem of a thin plate subjected to lateral loads falls into one of three categories: small or large elastic deformation or large deformation with inelastic material behavior. This study focuses on large elastic-plastic deformation problems of thin plates.

2.1.1 Small Elastic Deformations

In the range of small deformations ($w_{max} \leq .3t$) (2), the linear plate theory provides sufficiently accurate results. This theory assumes that strains in the middle surface of the plate are negligible, a straight line normal to the plate of the plate remains straight and normal when the plate is deformed, and stresses normal to the plane of the plate are zero.

The equilibrium equation for an infinitesimal element of an isotropic plate subjected to small deformations yields the following differential equation:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial^2 x \, \partial^2 y} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad (2.1)$$

where

q = intensity of the distributed load;

w = lateral deflection; and

$D = Et^3/12(1 - \nu^2)$, the bending stiffness of the plate.

For an orthotropic plate, the equilibrium formulation is given below:

$$D_x \frac{\partial^4 w}{\partial x^4} + (\nu_y D_x + \nu_x D_y + 4D_{xy}) \frac{\partial^4 w}{\partial x^2 \, \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q \qquad (2.2)$$

In Equation (2.2), ν_x and ν_y are Poisson's ratios associated with bending in the x and y directions (1); D_x, D_y, and D_{xy} are the orthotropic bending and twisting stiffnesses.

2.1.2 Large Elastic Deformations

When the lateral displacement of a plate exceeds three-tenths of its thickness (2), the small deformation theory no longer provides accurate results. In this range of deflection ($w \geq .3t$), the magnitude of the midplane strains and consequently the magnitude of the midsurface forces become significant. The vertical components of these membrane forces resist a portion of the lateral loads. The most common equilibrium formulation that includes this effect is known as the Von Karman equations (3).

The large deformation theory assumes that a straight line normal to the plane of the plate before bending remains straight and normal during bending and that the stresses normal to the plate of the plate are negligible. Figure 1 shows forces and moments on an infinitesimal element of an isotropic plate. The equations of equilibrium for this element in x and y directions are:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{dy} = 0 \tag{2.3a}$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \tag{2.3b}$$

The equilibrium equation for the infinitesimal element in the z direction becomes

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\,(q + N_x\,\frac{\partial^2 w}{\partial x^2} + N_y\,\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\,\frac{\partial^2 w}{\partial x \partial y}) \tag{2.4}$$

where

$$N_x = \frac{Et}{(1 - \nu^2)}\,[\frac{\partial u}{\partial x} + \frac{1 + \nu}{2}\,(\frac{\partial w}{\partial x})^2 + \nu\,\frac{\partial v}{\partial y}] \tag{2.5a}$$

$$N_y = \frac{Et}{(1 - \nu^2)}\,[\frac{\partial v}{\partial y} + \frac{1 + \nu}{2}\,(\frac{\partial w}{\partial y})^2 + \nu\,\frac{\partial u}{\partial x}] \tag{2.5b}$$

$$N_{xy} = Gt\,[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\,\frac{\partial w}{\partial y}] \tag{2.5c}$$

As seen in Equation (2.5), the membrane forces N_x, N_y, and N_{xy} are

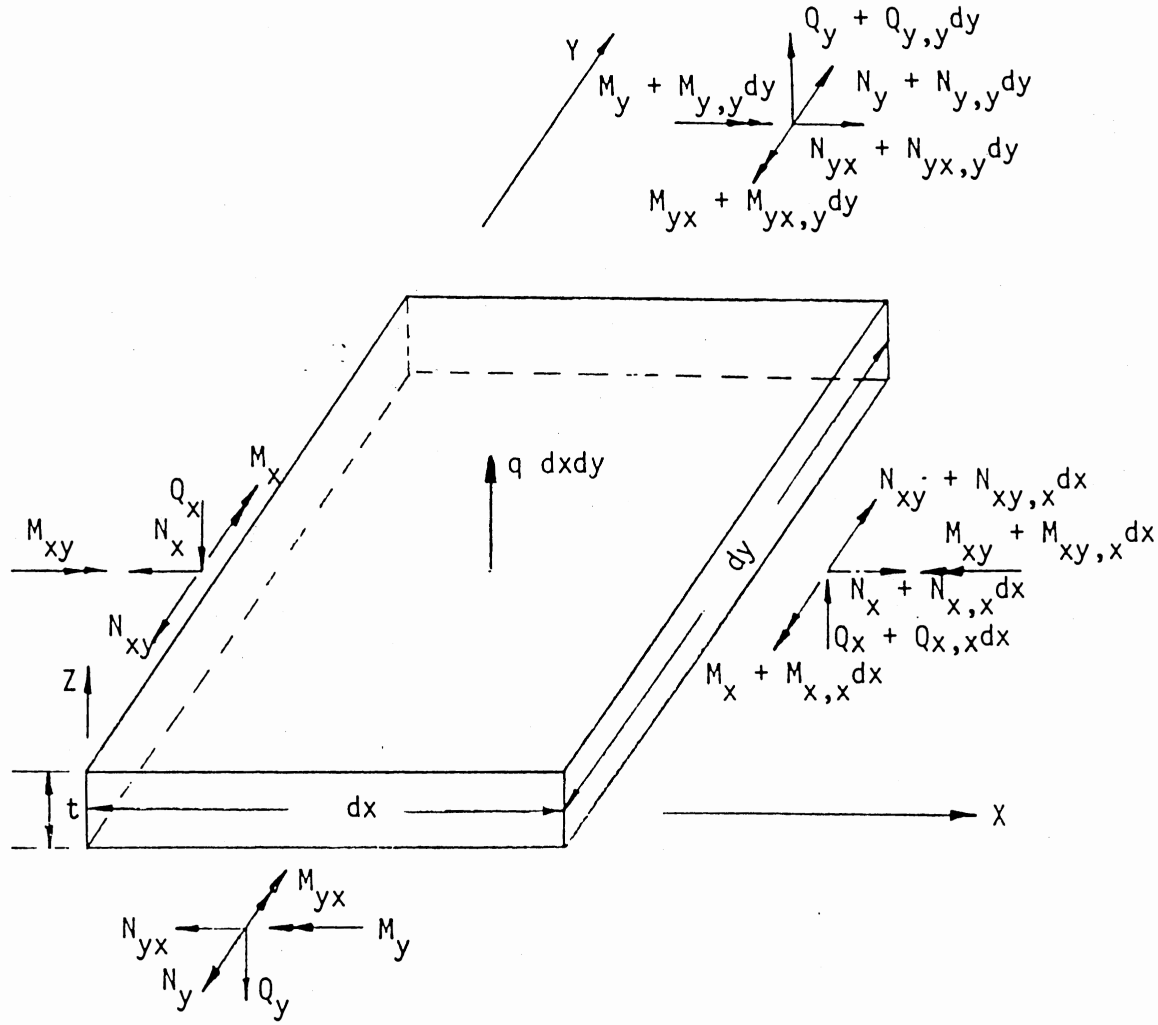


Figure 1. Forces on an Infinitesimal Element of a Plate (1)

related to the products and squares of the derivatives of plate deflections. This nonlinear relation is termed "geometric nonlinearity."

In the case of a large deflection of elastic orthotropic plates, Equation (2.4) can be modified to reflect the orthotropic material properties as follows:

$$D_x \frac{\partial^4 w}{\partial x^4} + (\nu_y D_x + \nu_x D_y + 4D_{xy}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4}$$

$$= q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} \tag{2.6}$$

where

$$D_x = \frac{E_x t^3}{12} (1 - \nu_x \nu_y) \tag{2.7a}$$

$$D_y = \frac{E_y t^3}{12} (1 - \nu_x \nu_y) \tag{2.7b}$$

$$D_{xy} = \frac{G_{xy} t^3}{12} \tag{2.7c}$$

and

$$N_x = \frac{E_x t}{(1 - \nu_x \nu_y)} \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x}\right)^2 + \nu_y \frac{\partial v}{\partial y} + \frac{\nu_y}{2} \left(\frac{\partial w}{\partial x}\right)^2 \right] \tag{2.8a}$$

$$N_y = \frac{E_y t}{(1 - \nu_x \nu_y)} \left[\frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y}\right)^2 + \nu_x \frac{\partial u}{\partial x} + \frac{\nu_x}{2} \left(\frac{\partial w}{\partial y}\right)^2 \right] \tag{2.8b}$$

$$N_{xy} = G_{xy}\, t \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}\right] \tag{2.8c}$$

Equation (2.6) is a nonlinear partial differential equation for which a closed-form solution is not available. A review of the different numerical approaches for solving this equation for isotropic and orothtropic plates appears at the end of this chapter.

2.1.3 Large Elastic-Plastic Deformations

The simultaneous occurrence of both geometric and material nonlinearities create additional problems for analysis of plates. Due to the absence of closed-form solutions for these highly nonlinear problems, recourse is taken to numerical methods. Numerical approaches such as the finite element or finite difference method simulate the behavior of the continuum. Equilibrium formulation for these models leads to simultaneous nonlinear equations which can be solved by numerical techniques.

2.2 Theory of Plasticity

At a point in an elastic continuum, stresses and strains are related to one another through a series of linear equations derived from a generalization of Hooke's Law (4). If the strains for this point are known, one can determine corresponding stresses through a one-step solution. The relation between stresses and corresponding strains at a point in a plastic continuum is nonlinear. If the strains for such a point are known, the corresponding stresses can be evaluated through a two-step solution:

1. Identification of the onset of the plastic flow at the point through application of one of the several yield criteria.

2. Application of the associated flow rule--i.e., the nonlinear constitutive law relating stresses, total strains, and increments of plastic strain.

Details of this two-step solution are given in the subsequent sections of this chapter.

2.2.1 Yield Criterion

Identification of the onset of plastic behavior at a point in a continuum is made possible by applying a yield criterion. This study uses the Von Mises yield criterion (5). This criterion assumes that yielding at a point subjected to a general state of stress occurs when the distortion energy is equal to the material's yield distortion energy found by a uniaxial tension test. The effective stress (σ_e) at a point is

$$\sigma_e = \frac{1}{\sqrt{2}} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{yz}^2 + \tau_{xz}^2)]^{1/2} \tag{2.9}$$

According to the Von Mises criterion, yielding begins in the triaxial state whenever the effective stress equals or exceeds yield stress of a tension specimen. Equation (2.9) represents a hypersurface in the six-dimensional Haig-Westergaard space (6). Any point on this surface is a point at which yield begins. To verify the validity of the Von Mises criterion, Lode (7) in 1925 and Taylor and Quinney (8) in 1931 ran a series of experiments on pressurized pipes subjected to axial forces. The results of these experiments within a certain range of stresses agree with the Von Mises theory.

2.2.2 Plastic Flow Constitutive Equations

The first formulation of plastic stress-strain relations were suggested by Saint-Venant (9) in 1870. This work was extended by Lévy (10) and Von Mises (5). Prandtl and Reuss modified the work of Lévy and Von Mises to arrive at what is referred to as the Prandtl-Reuss equations (5). A summary of the derivation of the Prandtl-Reuss equations is included to provide the fundamental background for the equations used in this study.

The state of stress at a point in a body in the plastic range can be defined by

$$\sigma_{x,i},\ \sigma_{y,i},\ \sigma_{z,i},\ \tau_{xy,i},\ \tau_{yz,i},\ \tau_{xz,i}$$

and, as a result of a change in loads, the state of stress at the same point becomes

$$\sigma_{x,i+1},\ \sigma_{y,i+1},\ \sigma_{z,i+1},\ \tau_{xy,i+1},\ \tau_{yz,i+1},\ \tau_{xz,i+1}$$

Associated with the change in stress, plastic strain increments are produced. The plastic strain increments are

$$\Delta\varepsilon^{p}_{x,i+1},\ \Delta\varepsilon^{p}_{y,i+1},\ \Delta\varepsilon^{p}_{z,i+1},\ \Delta\varepsilon^{p}_{xy,i+1},\ \Delta\varepsilon^{p}_{yz,i+1},\ \Delta\varepsilon^{p}_{xz,i+1}$$

The plastic strain increments satisfy the material incompressibility condition in the plastic range

$$\Delta\varepsilon^{p}_{x,i+1} + \Delta\varepsilon^{p}_{y,i+1} + \Delta\varepsilon^{p}_{z,i+1} = 0$$

Prandtl and Reuss assume that the plastic strain increment, at any instant of loading, is proportional to the instantaneous stress deviation and shear stresses; that is,

$$\frac{\Delta\varepsilon^{p}_{x,i+1}}{S_{x,i+1}} = \frac{\Delta\varepsilon^{p}_{y,i+1}}{S_{y,i+1}} = \frac{\Delta\varepsilon^{p}_{z,i+1}}{S_{z,i+1}} = \frac{\Delta\varepsilon^{p}_{xy,i+1}}{\tau_{xy,i+1}}$$

$$= \frac{\Delta\varepsilon^{p}_{xz,i+1}}{\tau_{xz,i+1}} = \frac{\Delta\varepsilon^{p}_{yz,i+1}}{\tau_{yz,i+1}} = d\lambda \qquad (2.10)$$

where

$$S_{x,i+1} = [(2\sigma_x - \sigma_y - \sigma_z)/3]_{i+1} \tag{2.11a}$$

$$S_{y,i+1} = [(2\sigma_y - \sigma_x - \sigma_z)/3]_{i+1} \tag{2.11b}$$

$$S_{z,i+1} = [(2\sigma_z - \sigma_x - \sigma_y)/3]_{i+1} \tag{2.11c}$$

and $d\lambda$ is a constant at each loading stage.

In this formulation it may be demonstrated that

$$\sigma_{e,i+1} = \frac{1}{\sqrt{2}} [(\sigma_x - \sigma_y)^2_{i+1} + (\sigma_x - \sigma_z)^2_{i+1} + (\sigma_y - \sigma_z)^2_{i+1} + 6(\tau^2_{xy} + \tau^2_{xz} + \tau^2_{yz})_{i+1}]^{1/2} \tag{2.12}$$

and that the increment of equivalent plastic strain is

$$\Delta\varepsilon^p_{i+1} = \frac{\sqrt{2}}{3} [(\Delta\varepsilon^p_x - \Delta\varepsilon^p_y)^2_{i+1} + (\Delta\varepsilon^p_y - \Delta\varepsilon^p_z)^2_{i+1} + (\Delta\varepsilon^p_x - \Delta\varepsilon^p_z)^2_{i+1} + 6(\Delta\varepsilon^{p2}_{xy} + \Delta\varepsilon^{p2}_{xz} + \Delta\varepsilon^{p2}_{yz})_{i+1}]^{1/2} \tag{2.13}$$

The relation between $\sigma_{e,i+1}$ and $\Delta\varepsilon^p_{i+1}$ may be expressed mathematically as

$$d\lambda = \frac{3\Delta\varepsilon^p_{i+1}}{2\sigma_{e,i+1}} \tag{2.14}$$

For a linear work-hardening material, the relation between these quantities is shown in Figure 2.

At the stage of loading designated as $i + 1$, we can state that the total strain in any direction is equal to the sum of the elastic and plastic strain increments caused by the increase in load from i to $i + 1$.

$$\varepsilon_{x,i+1} = \frac{1}{E_o} [\sigma_x - \nu_o (\sigma_y + \sigma_z)]_{i+1} + \sum_{k=1}^{i} \Delta\varepsilon^p_{x,k} + \Delta\varepsilon^p_{x,i+1} \tag{2.15a}$$

$$\varepsilon_{y,i+1} = \frac{1}{E_o} [\sigma_y - \nu_o (\sigma_z + \sigma_x)]_{i+1} + \sum_{k=1}^{i} \Delta\varepsilon^p_{y,k} + \Delta\varepsilon^p_{y,i+1} \tag{2.15b}$$

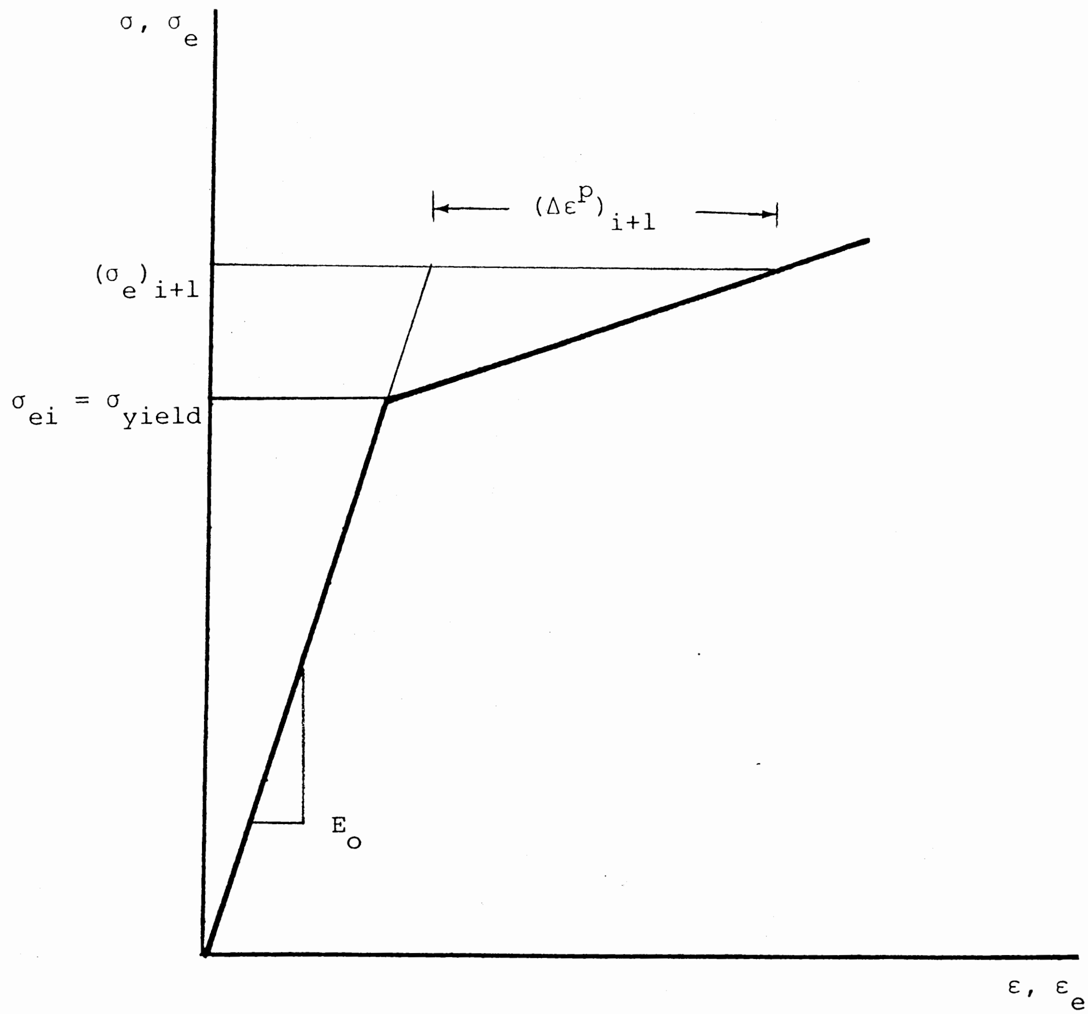


Figure 2. Equivalent Plastic Strain Increment for Linear Work-Hardening Material

$$\varepsilon_{z,i+1} = \frac{1}{E_o} [\sigma_z - \nu_o (\sigma_x + \sigma_y)]_{i+1} + \sum_{k=1}^{i} \Delta\varepsilon^p_{z,k} + \Delta\varepsilon^p_{z,i+1} \tag{2.15c}$$

$$\varepsilon_{xy,i+1} = \frac{1}{2G_o} \tau_{xy,i+1} + \sum_{k=1}^{i} \Delta\varepsilon^p_{xy,k} + \Delta\varepsilon^p_{xy,i+1} \tag{2.15d}$$

$$\varepsilon_{yz,i+1} = \frac{1}{2G_o} \tau_{yz,i+1} + \sum_{k=1}^{i} \Delta\varepsilon^p_{yz,k} + \Delta\varepsilon^p_{yz,i+1} \tag{2.15e}$$

$$\varepsilon_{xz,i+1} = \frac{1}{2G_o} \tau_{xz,i+1} + \sum_{k=1}^{i} \Delta\varepsilon^p_{xz,k} + \Delta\varepsilon^p_{xz,i+1} \tag{2.15f}$$

The left side of these equations shows the total strains at the point. E_o is the slope of the elastic portion of the stress-strain curve of the material in pure tension (Figure 2), and ν_o is the elastic Poisson's ratio. The modulus of rigidity for the material in the elastic range is G_o. Equations (2.10) through (2.13) and the magnitude of $d\lambda$ can be combined to yield

$$\Delta\varepsilon^p_{x,i+1} = \left\{\frac{\Delta\varepsilon^p}{\sigma_e} [\sigma_x - \frac{1}{2} (\sigma_y + \sigma_z)]\right\}_{i+1} \tag{2.16a}$$

$$\Delta\varepsilon^p_{y,i+1} = \left\{\frac{\Delta\varepsilon^p}{\sigma_e} [\sigma_y - \frac{1}{2} (\sigma_x + \sigma_z)]\right\}_{i+1} \tag{2.16b}$$

$$\Delta\varepsilon^p_{z,i+1} = \left\{\frac{\Delta\varepsilon^p}{\sigma_e} [\sigma_z - \frac{1}{2} (\sigma_x + \sigma_y)]\right\}_{i+1} \tag{2.16c}$$

$$\Delta\varepsilon^p_{xy,i+1} = \left\{\frac{3\Delta\varepsilon^p}{2\sigma_e} (\tau_{xy})\right\}_{i+1} \tag{2.16d}$$

$$\Delta\varepsilon^p_{yz,i+1} = \left\{\frac{3\Delta\varepsilon^p}{2\sigma_e} (\tau_{yz})\right\}_{i+1} \tag{2.16e}$$

$$\Delta\varepsilon^p_{xz,i+1} = \left\{\frac{3\Delta\varepsilon^p}{2\sigma_e} (\tau_{xz})\right\}_{i+1} \tag{2.16f}$$

For the special case of plane stress, these equations reduce to the following equations:

$$\varepsilon_{x,i+1} = \left[\frac{1}{E_o}(\sigma_x - \nu_o\sigma_y)\right]_{i+1} + \sum_{k=1}^{i} \Delta\varepsilon^p_{x,k} + \Delta\varepsilon^p_{x,i+1} \tag{2.17a}$$

$$\varepsilon_{y,i+1} = \left[\frac{1}{E_o}(\sigma_y - \nu_o\sigma_x)\right]_{i+1} + \sum_{k=1}^{i} \Delta\varepsilon^p_{y,k} + \Delta\varepsilon^p_{y,i+1} \tag{2.17b}$$

$$\varepsilon_{xy,i+1} = \left[\frac{1}{2G_o}\tau_{xy}\right]_{i+1} + \sum_{k=1}^{i} \Delta\varepsilon^p_{xy,k} + \Delta\varepsilon^p_{xy,i+1} \tag{2.17c}$$

and

$$\Delta\varepsilon^p_{x,i+1} = \left[\frac{\Delta\varepsilon^p}{2\sigma_e}(2\sigma_x - \sigma_y)\right]_{i+1} \tag{2.18a}$$

$$\Delta\varepsilon^p_{y,i+1} = \left[\frac{\Delta\varepsilon^p}{2\sigma_e}(2\sigma_y - \sigma_x)\right]_{i+1} \tag{2.18b}$$

$$\Delta\varepsilon^p_{xy,i+1} = \left[\frac{3\Delta\varepsilon^p}{2\sigma_e}(\tau_{xy})\right]_{i+1} \tag{2.18c}$$

For the special case when $\sigma_{e,i} = \sigma_{yield}$ and $\sigma_{e,i+1}$ is on the second branch of the stress-strain curve as shown in Figure 2, Equations (2.17) and (2.18) may be written as

$$\varepsilon_{x,i+1} = \left[\frac{1}{E_o}(\sigma_x - \nu_o\sigma_y)\right]_{i+1} + \Delta\varepsilon^p_{x,i+1} \tag{2.19a}$$

$$\varepsilon_{y,i+1} = \left[\frac{1}{E_o}(\sigma_y - \nu_o\sigma_x)\right]_{i+1} + \Delta\varepsilon^p_{y,i+1} \tag{2.19b}$$

$$\varepsilon_{xy,i+1} = \left[\frac{1}{2G_o}\tau_{xy\ i+1}\right] + \Delta\varepsilon^p_{xy,i+1} \tag{2.19c}$$

and

$$\Delta\varepsilon^p_{x,i+1} = \left[\frac{\Delta\varepsilon^p}{2\sigma_e}(2\sigma_x - \sigma_y)\right]_{i+1} \tag{2.20a}$$

$$\Delta\varepsilon^p_{y,i+1} = \left[\frac{\Delta\varepsilon^p}{2\sigma_e}(2\sigma_y - \sigma_x)\right]_{i+1} \tag{2.20b}$$

$$\Delta\varepsilon^p_{xy,i+1} = \left[\frac{3\Delta\varepsilon^p}{2\sigma_e}(\tau_{xy})\right]_{i+1} \tag{2.20c}$$

Equations (2.13), (2.19), and (2.20) are used in this study for the analysis of plates. The application of these equations will be fully discussed in Chapter III.

2.3 Review of Numerical Methods

Investigators have used various numerical techniques for calculating the large deflections and accompanying stresses of plates in the elastic and elastic-plastic range of material behavior. To demonstrate the capability of solution methods, they have solved some example problems and have compared their results with available experimental data or analytical results. A survey of both analytical and experimental work related to large deflection of plates is presented in the following sections.

2.3.1 Elastic Deformations

Equilibrium formulation for elastic plates subjected to large transverse displacements results in nonlinear partial differential equations (the Von Karman equations) for which limited closed-form solutions are available for specific problems. For the general problem, numerical techniques are suggested by the following researchers. Levy (11) uses the double Fourier series for analyzing a group of rectangular thin plates having a length to width ratio of 1.5 and subjected to uniform lateral pressure. The accuracy of Levy's solution can be enhanced to any desired degree by increasing the number of terms in his solution. Wang (12) uses a finite difference approach coupled with a method of successive approximation to solve the Von Karman equations. With the same objective, Way (13) applies the Ritz Energy Method and uses polynomials that satisfy the plate boundary conditions. Chien and Yeh (14) use the perturbation procedure to solve nonlinear plate problems while Kaiser (15) converts the Von Karman equations into nonlinear finite difference equations and applies them to the problem of a simply supported plate with movable edges. In his solution he decomposes his biharmonic operator into two Laplace operators. Most of these examples are studied for a uniformly lateral loading. Vallahban (16) uses the Von Karman equations to analyze the behavior of simply supported thin rectangular glass plates subjected to lateral pressure. The equations are expanded by utilizing finite difference form. An iterative technique using a nonlinear interpolation factor is used for solving the difference equations. Murray and Wilson (17) use a finite element technique to solve for the Von Karman equations. Leesavan (1) converts the equilibrium and compatibility equations of the node points of a mathematically consistent model into finite difference equations and solves for plate deflections. To verify their method of analysis, many researchers report a comparison with analytical work developed by others. In the present study (Chapter VI) results of the analysis of large elastic deflection of a thin square plate are given and compared with those obtained by Vallahban.

2.3.2 Elastic-Plastic Deformations

In evaluating the solution procedure for material and geometrically nonlinear behavior, Stricklin et al. (18) write that

> Although a great number of papers here are published on the analysis of nonlinear behavior, a thorough survey of this available literature indicates that there is considerable uncertainty regarding such important questions as which yield criteria are the best, which flow rule is correct, how should unloading be treated, and so on. Perhaps the most perplexing and disturbing point has been that most of the computational procedures currently available are very inefficient and require excessive amount of computer time when applied to practical large scale structural systems (p. 292).

Analytical solutions for the problem of the large deflection of elastic-plastic circular plates are abundant in the literature. Cross and Ang (19) use the discrete-element model to solve for deflections of circular plates in the elastic-plastic range. In the solution procedure suggested by Onat and Haythornthwaite (20), the load-carrying capacity after finite deflections is estimated by assuming a velocity field based on boundary conditions and on the incipient velocity field of the flat plate. This analysis is made for a rigid-plastic nonstrain hardening material that yields according to the maximum shear-stress criterion. Ohashi and Kamiya (21) suggest a solution in which the stress-strain relation is expressed by a simple power function.

Analytical solutions for the problem of large deflection of rectangular plates in the elastic-plastic range are less abundant. Marcal (22), Armen and Pifko (23), and Murray and Wilson (24) are among those who suggest analytical methods for solving this type of problem. Murray and Wilson, for example, use the finite element technique. Their approach to a solution is incremental and iterative and couples stretching and bending behaviors. For each increment of loading, tangent stiffness is computed and used for all iterations within the increment. The convergence to equilibrium is tested, however, by utilizing secant stiffness computed after each iteration. Recent studies include the work of Kuo-Kuang (25). This author uses a simple, nonlinear triangular finite element model. Bathe and Bolourchi (26) also use a finite element technique. Both references utilize the Lagrangian strain-displacement relationship. A detailed survey and a tabulated list of the current computer programs for the large deflection of structures in the elastic-plastic range and the capability of these programs appears in a recent work by Noor (27).

2.3.3 Experimental Data

A large amount of test data is available to verify the analytical solution of the large deflection of circular plates. While the majority of experimental work is limited to the development of load-deflection characteristics of circular plates, some reports include such features as strain and curvature distribution. Tests carried out by Ramberg, McPherson, and Levy (28) include residual deformation and strain measurement of aluminum, magnesium, and stainless steel plates. Tests made by Ohashi and Murakami (29) and Ohashi and Kawashima (30) evaluate the strain distribution in a circular plate. The work of Cooper and Shirfin (31) reports on strain and curvature of mild steel circular plates near the edges. Sherbourne and Srivastra (32) perform experiments to demonstrate the complete spectrum of plate behavior from yielding due to bending action, increased stiffness due to membrane action, and, finally, plastic softening of the membrane resistance.

The earliest recorded tests on a clamped rectangular plate were run for the Russian Navy in 1902 (33) and the German Navy in 1911 (33). The most notable tests on the large deflection of plates are two series of tests sponsored by the National Advisory Committee for Aeronautics (NACA) and conducted by Ramberg, McPherson, and Levy (27) in 1942. The lack of perfect clamping in these tests led to inconsistent results. When Clarkson (34) and Young (35) suggested their elastic-plastic theories of the large deflection of plates in 1956 and 1959, respectively, no data on the elastic-plastic behavior of mild steel plates were available for the purpose of comparison. Both authors performed confirmatory rests. Hooke and Rawlings (33) ran a series of tests on clamped thin mild steel plates subjected to lateral pressure. They admit to inaccuracies in the test results, but also suggest methods for correcting the errors. The plate structure used by these investigators is analyzed by the method presented in this report in which analytical results are compared with the experimental data. Agreement with experimental data produces a degree of confidence that the method of analysis converges to correct values of deformation and stress.

CHAPTER III

THE DISCRETE-ELEMENT MODEL

Analysis of many problems of continuum mechanics requires the solution of nonlinear partial differential equations for which closed-form solutions are not available. An approach for solving such problems is the replacement of partial differential equations with finite difference equations. The finite difference equations are then related to mathematically consistent, discrete-element (lumped-parameter) models that are point-by-point compatible with the continuum. This type of procedure for solving plate problems is suggested by Newmark and later extended by Ang (36).

For the large deflection of plates, membrane as well as bending effects help resist the transverse loads. The discrete-element model used for this study has two components--a bending and a membrane component. The properties of the discrete-element model used in this study are similar to those used by Leesavan (1) for solving the geometrically nonlinear problem of elastic large deflections. The present work extends Leesavan's work to include the effect of material nonlinearity.

3.1 The Plate Bending Model

The bending component of the plate model, which is also referred to as the bending model, is shown in Figure 3. This model resists lateral loads through bending action. The configuration and properties of the

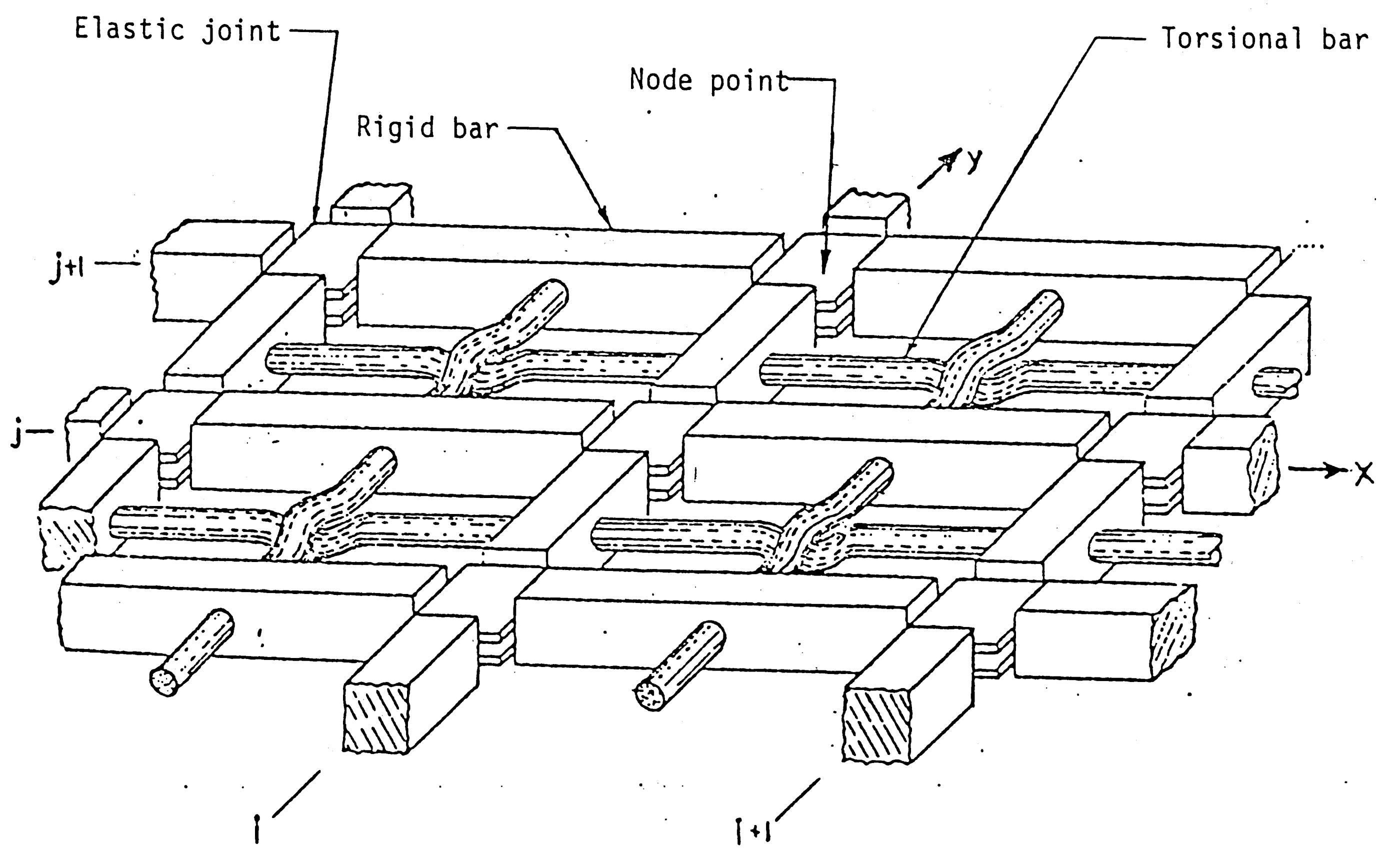


Figure 3. Discrete Element Bending Model (1)

bending model are similar to those of Hudson et al. (37), and later those of Kelly (38) and Leesavan (1) for the study of nonlinear plate problems.

The bending model shown in Figure 3 is made up of elements that represent differing aspects of the bending resistance capability of the plate. Rigid telescoping bars connect the nodes of the model and resist lateral shear. The torsional stiffness of the plate is represented by two bars in each panel. The bending stiffness of the plate, related to geometric and material properties, is assumed to be concentrated at the flexible nodes. The relation between joint flexibility and plate properties is presented in Appendix B. Although the derivation in Appendix B is for an elastic model, the form of the equations is unchanged for the inelastic problem. In the plastic range, each solution is performed by utilizing an equivalent elastic modulus of elasticity for bending.

3.2 The Plate Membrane Model

The membrane component of the plate model, also referred to as the membrane model, is similar to the one used by Leesavan (1) and developed by Herennikoff (39) so that a plate with in-plane loading may be replaced by a truss structure. As the result of out-of-plane deformations, the membrane forces produce resistance to the lateral loads. The membrane model is made of flexible bars connected by socket joints (Figure 4). Properties of these bars are determined from the geometric and material properties of the plate. A detailed explanation of this model and the equations used are presented in Appendix C. Although the equations are initially derived for plates with elastic material properties, they are applied in the present study for analyzing plates with inelastic material

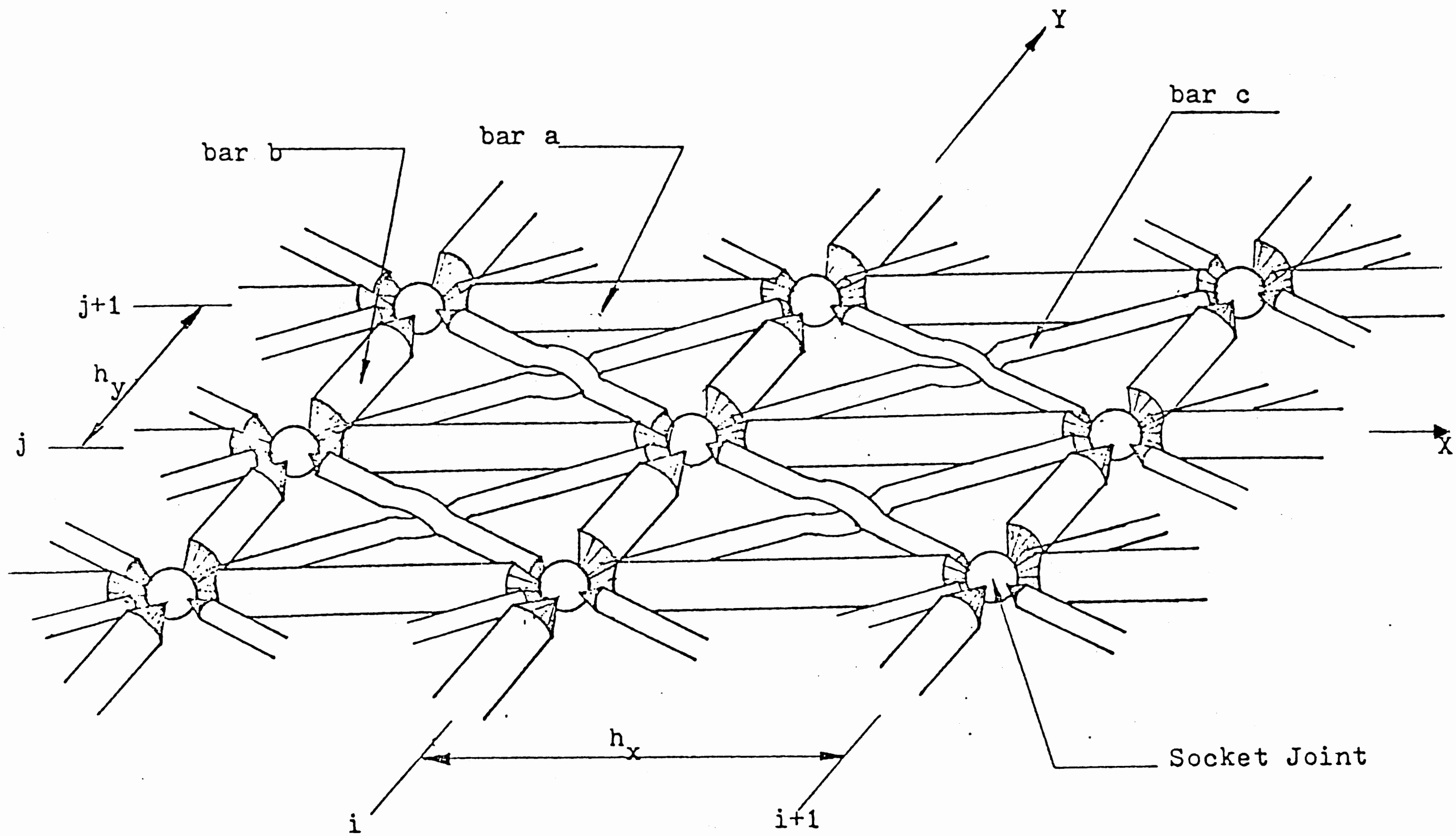


Figure 4. Discrete Element Membrane Model (1)

properties. In the inelastic range the analysis is performed by utilizing equivalent elastic material constants.

3.3 The Composite Model

The composite model is composed of the bending and membrane models connected by means of transverse rigid bars at nodes as shown in Figure 5. The composite system resists both lateral and in-plane loads. When total loads are applied to the structure, the initial resistance is provided by the bending resistance of the system. As a result of vertical nodal displacement, axial forces may be produced in the elements of the membrane model. The horizontal component of these bar forces at the nodes of the membrane model will usually not be in equilibrium. Due to this imbalance of forces, the nodes of the membrane model move in the horizontal plane until equilibrium is established in the model. In this configuration the load-carrying capacity of the composite model is equal to the resistance provided by the bending model plus that provided by the vertical components of membrane bar forces. To achieve equilibrium, repeated solutions are performed until both transverse and in-plane equilibrium is satisfied at each point.

3.4 Model Strain Calculations

A section of the plate surrounding each node point is assumed to be constructed from a number of plate-like elements of thickness Δz which are separated from one another by frictionless surfaces as shown in Figure 6. Each lamina of the structure is subjected to a state of strain consistent with the joint deformations. This state of strain is used to assess the state of stress and, subsequently, the stress resultants in

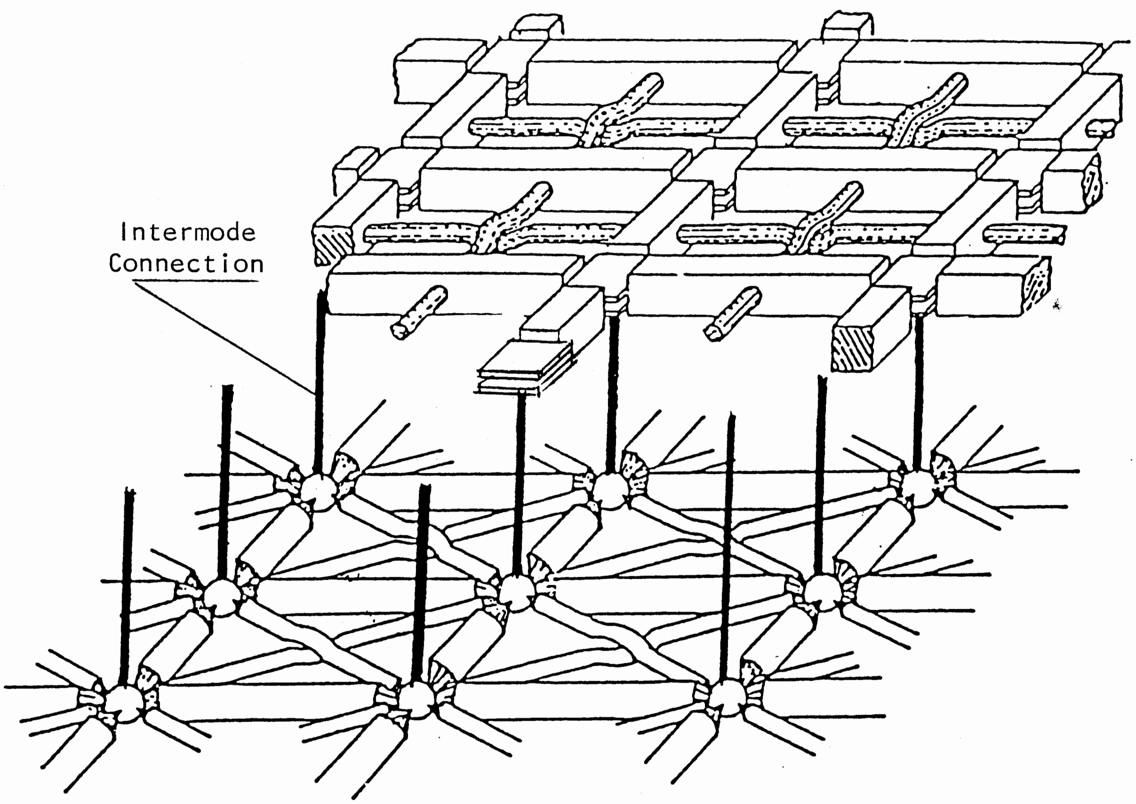
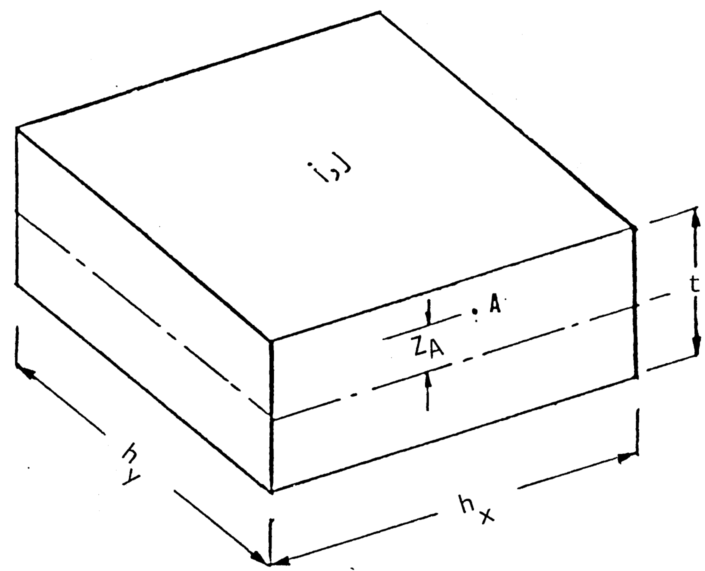
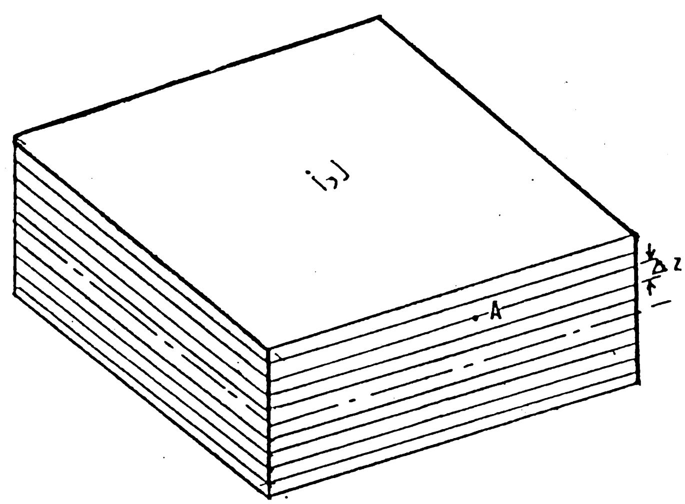


Figure 5. Composite Model



(a) Geometry of the Region



(b) Plate Lamina

Figure 6. Plate Region for Node i,j

the plate. The stress resultants are related to curvature and midplane strains by equivalent elastic material properties. In the general problem each node point may have a unique set of equivalent elastic properties, one for bending and another for the plane stress problem of in-plane loading.

3.4.1 Finite Difference Equations for Curvature and Midsurface Strain

Expressions for curvatures at a point on the midsurface of a plate are

$$\phi_x = \frac{(\partial^2 w/\partial^2 x)}{[1 + (\partial w/\partial x)^2]^{3/2}} \tag{3.1a}$$

$$\phi_y = \frac{(\partial^2 w/\partial^2 y)}{[1 + (\partial w/\partial y)^2]^{3/2}} \tag{3.1b}$$

$$\phi_{xy} = \frac{\partial^2 w}{\partial x \partial y} \tag{3.1c}$$

Expressions for plate midsurface strains at the same point are

$$\varepsilon_{xm} = \left(\frac{\partial u}{\partial x}\right) + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{3.2a}$$

$$\varepsilon_{ym} = \left(\frac{\partial v}{\partial y}\right) + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \tag{3.2b}$$

$$\varepsilon_{xym} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} \tag{3.2c}$$

For point-by-point compatibility between the continuum and the model, the magnitude of the curvatures and the midsurface strains are related by finite difference approximations. With reference to the node identification shown in Figure 7, curvatures at node i,j can be written as follows:

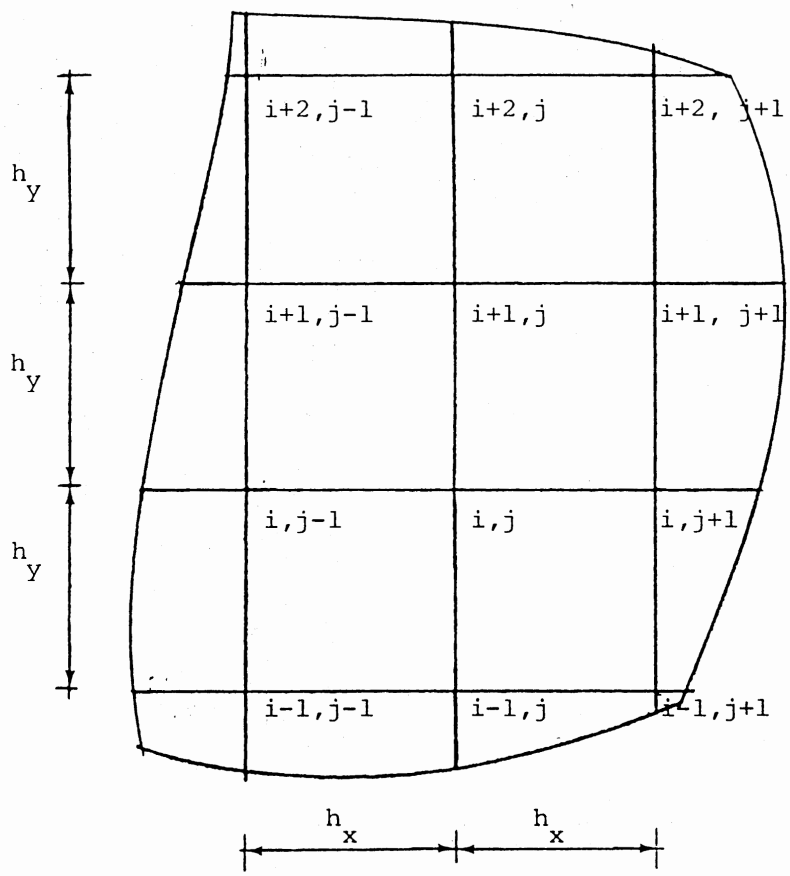


Figure 7. Arrangement of Nodes

$$(\phi_x)_{i,j} = \frac{[w_{i-1,j} - 2w_{i,j} + w_{i+1,j}]}{h_x^2\left[1+\left(\frac{w_{i+1,j} - w_{i-1,j}}{2h_x}\right)^2\right]^{3/2}} \tag{3.3a}$$

$$(\phi_y)_{i,j} = \frac{[w_{i,j-1} - 2w_{i,j} + w_{i,j+1}]}{h_y^2\left[1+\left(\frac{w_{i,j+1} - w_{i,j-1}}{2h_y}\right)^2\right]^{3/2}} \tag{3.3b}$$

$$(\phi_{xy})_{i,j} = \frac{[w_{i+1,j+1} + w_{i-1,j-1} - w_{i-1,j+1} - w_{i+1,j-1}]}{4h_x \cdot h_y} \tag{3.3c}$$

The average midsurface strains for a node i,j of the membrane model can also be written as follows:

$$(\varepsilon_{xm})_{i,j} = \frac{1}{2h_x}[u_{i+1,j} - u_{i-1,j}] + \frac{1}{4h_x^2}[(w_{i+1,j} - w_{i,j})^2 + (w_{i,j} - w_{i-1,j})^2] \tag{3.4a}$$

$$(\varepsilon_{ym})_{i,j} = \frac{1}{2h_y}[v_{i,j+1} - v_{i,j-1}] + \frac{1}{4h_y^2}[(w_{i,j+1} - w_{i,j})^2 + (w_{i,j} - w_{i,j-1})^2] \tag{3.4b}$$

$$(\varepsilon_{xym})_{i,j} = \frac{1}{2h_x}[v_{i+1,j} - v_{i-1,j}] + \frac{1}{2h_y}[u_{i,j+1} - u_{i,j-1}] + \frac{1}{4h_x h_y}[w_{i+1,j} - w_{i-1,j}][w_{i,j+1} - w_{i,j-1}] \tag{3.4c}$$

The normal and shearing strains at a node of the model are assumed to be the average of the corresponding strains in the adjoining bars and panels.

3.4.2 Lamina Strains

In the previous section, curvatures and midsurface strains for node i,j of the plate were calculated. These quantities can be utilized to determine the state of strain at lamina A (Figure 6a) at this node as shown below:

$$(\varepsilon_x)^A_{i,j} = (\varepsilon_{xm})_{i,j} + Z_A\ (\phi_x)_{i,j} \tag{3.5a}$$

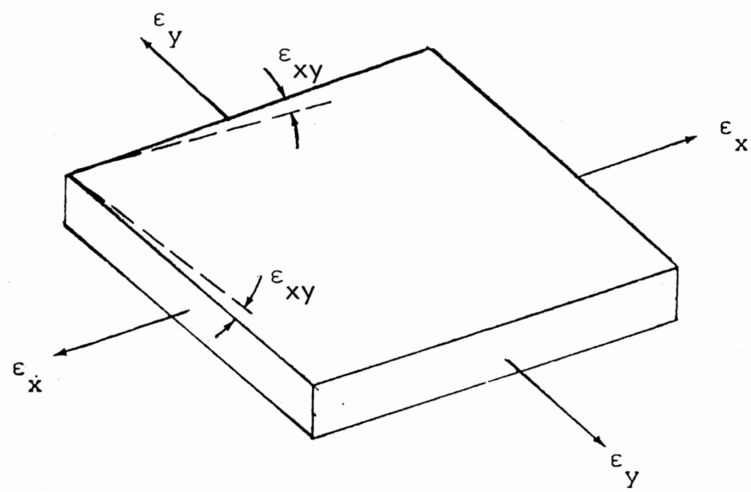
$$(\varepsilon_y)^A_{i,j} = (\varepsilon_{ym})_{i,j} + Z_A\ (\phi_y)_{i,j} \tag{3.5b}$$

$$(\varepsilon_{xy})^A_{i,j} = \frac{1}{2}\ (\varepsilon_{xym})_{i,j} + Z_A\ (\phi_{xy})_{i,j} \tag{3.5c}$$

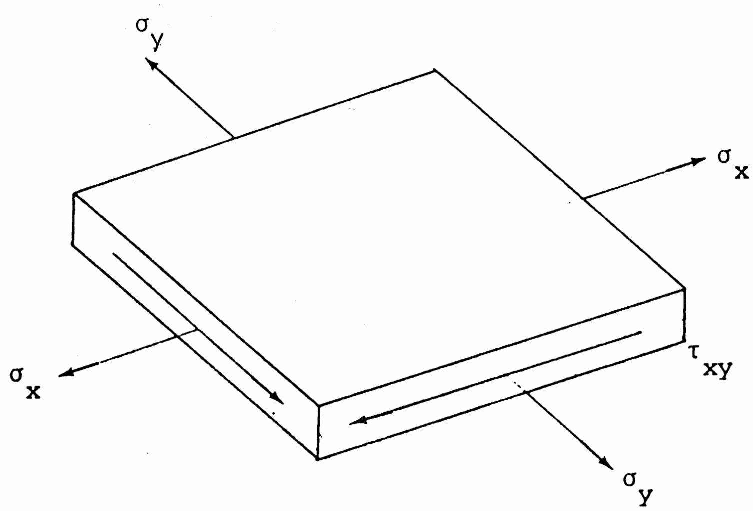
In Equation (3.5), $(\varepsilon_x)^A_{i,j}$, $(\varepsilon_y)^A_{i,j}$, and $(\varepsilon_{xy})^A_{i,j}$ are strains on lamina A; Z_A is the distance of this lamina from the midplane of the plate.

3.5 Lamina Stresses

In the previous section, the state of strain on a lamina A of the plate at node i,j was calculated. The objective of this section is to utilize the state of strain (Figure 8a), the stress-strain behavior of the material, and the relation between stresses, strains, and increments of plastic strain (Equations [2.13], [2.19], and [2.20]) to calculate the state of stress on the lamina (Figure 8b). Initially, the effective stress and strain are equal to zero and the secant material properties are equal to the initial elastic properties. When the plate is subjected to load, the state of strain may produce either elastic or inelastic material behavior. For the elastic case, Equation (2.19) with $\Delta\varepsilon^p_{x,i+1} = \Delta\varepsilon^p_{y,i+1} = \Delta\varepsilon^p_{xy,i+1} = 0$ is used for calculating the state of stress. For



(a) Strains on Plate Lamina



(b) Stresses on Plate Lamina

Figure 8. Lamina Stresses and Strains

the inelastic case, the lamina stresses are calculated by utilizing an iterative scheme. Initially, the magnitudes of increments of plastic strain ($\Delta\varepsilon^p_{x,i+1}$, $\Delta\varepsilon^p_{y,i+1}$, and $\Delta\varepsilon^p_{xy,i+1}$) are assumed to be 1000 times less than the strains ($\varepsilon_{x,i+1}$, $\varepsilon_{y,i+1}$, and $\varepsilon_{xy,i+1}$), respectively. By inserting the estimated quantities into Equations (2.19) and (2.13), the magnitudes of corresponding lamina stresses and increment of equivalent plastic strain ($\Delta\varepsilon^p_{i+1}$) are respectively calculated. Using the calculated magnitude of $\Delta\varepsilon^p_{i+1}$, the value of the effective stress ($\sigma_{e,i+1}$) is obtained from the effective stress-effective strain curve (Figure 9). The calculated quantitites are inserted into Equation (2.20) and new estimated values for the increments of plastic strain are determined. Convergence to true lamina stress is achieved whenever the difference between the magnitudes of increments of plastic strain in two successive iterations is less than a given tolerance. At this point, the effective stress, effective strain, and secant material properties of the lamina are utilized to define a new effective stress-effective strain curve (dashed line curve in Figure 9). The latter quantities are used for calculating the lamina stresses due to the next state of strain.

3.6 Stress Resultants

From the displacement field at each loading step, the strains and subsequently the stresses at each lamina of the plate are determined. The following equations are used to evaluate the stress resultants over the thickness of the plate segment of Figure 6a.

$$(\bar{N}_x)_{i,j} = \sum_{k=1}^{N} [(\sigma_x)^k_{i,j} \; \Delta z \; h_y] \tag{3.6a}$$

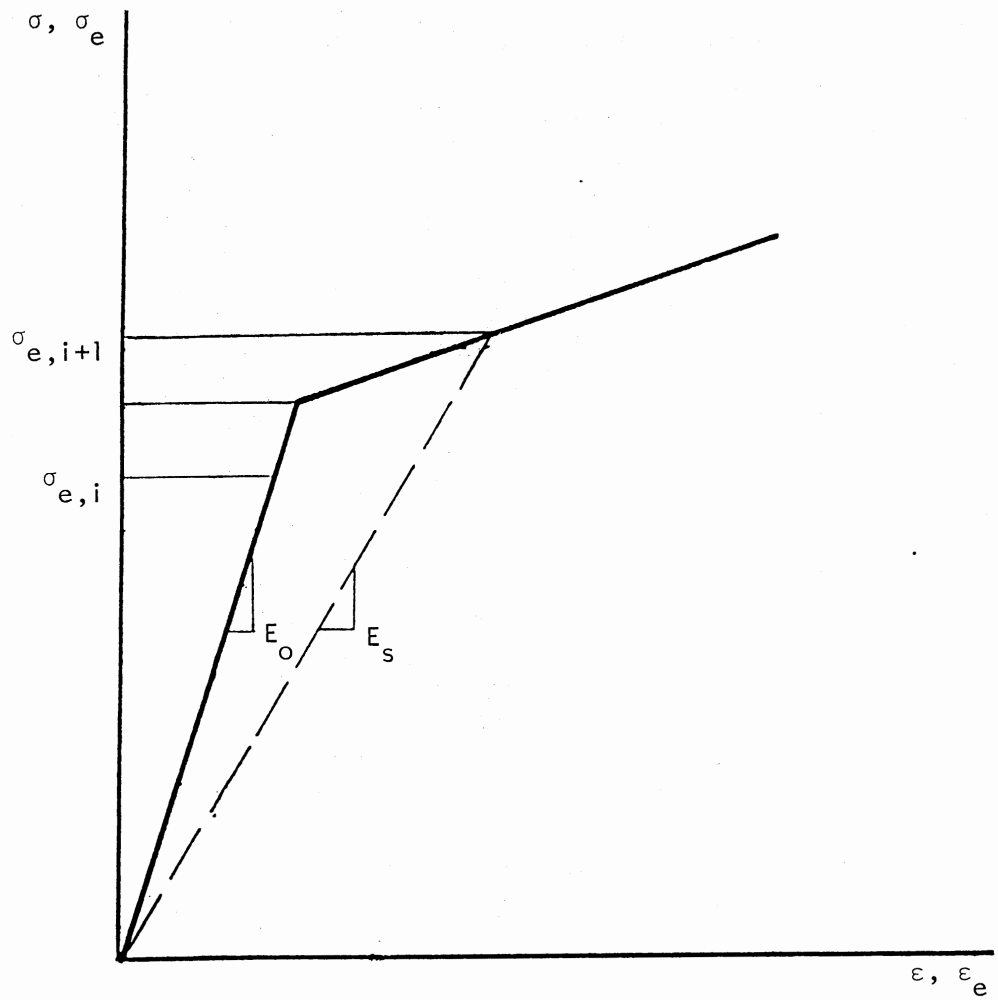


Figure 9. Effective Stress-Effective Strain Diagram for Plate Segment of Figure 7

$$(\bar{N}_y)_{i,j} = \sum_{k=1}^{N} [(\sigma_y)^k_{i,j} \; \Delta z \; h_x] \tag{3.6b}$$

$$(\bar{N}_{xy})_{i,j} = \sum_{k=1}^{N} [(\tau_{xy})^k_{i,j} \; \Delta z \; h_y] \tag{3.6c}$$

where $(\bar{N}_x)_{i,j}$, $(\bar{N}_y)_{i,j}$, and $(\bar{N}_{xy})_{i,j}$ are the resultant forces at node i,j; $(\sigma_x)^k_{i,j}$, $(\sigma_y)^k_{i,j}$, and $(\tau_{xy})^k_{i,j}$ are the normal and shearing stresses on the kth lamina at this node; and N equals the total number of laminae used to represent the plate. The resultant moments $(\bar{M}_x)_{i,j}$, $(\bar{M}_y)_{i,j}$, and $(\bar{M}_{xy})_{i,j}$ of the plate segment shown in Figure 6a are calculated as follows:

$$(\bar{M}_x)_{i,j} = \sum_{k=1}^{N} [(\sigma_x)^k_{i,j} \; (\frac{N}{2} - k) \; \Delta z^2 \; h_y] \tag{3.7a}$$

$$(\bar{M}_y)_{i,j} = \sum_{k=1}^{N} [(\sigma_y)^k_{i,j} \; (\frac{N}{2} - k) \; \Delta z^2 \; h_x] \tag{3.7b}$$

$$(\bar{M}_{xy})_{i,j} = \sum_{k=1}^{N} [(\sigma_{xy})^k_{i,j} \; (\frac{N}{2} - k) \; \Delta z^2 \; h_y] \tag{3.7c}$$

The stress distribution over the thickness of the plate at node i,j is shown in Figure 10. The stress resultants at this node are shown in Figure 11.

3.7 Equivalent Elastic Material Properties

In this study, it is assumed that the stress resultants calculated in the previous section are produced by pseudo-elastic material properties operating on curvatures and midsurface stretching. For node i,j the curvatures $((\phi_x)_{i,j}$, $(\phi_y)_{i,j}$, and $(\phi_{xy})_{i,j})$ and resultant moments

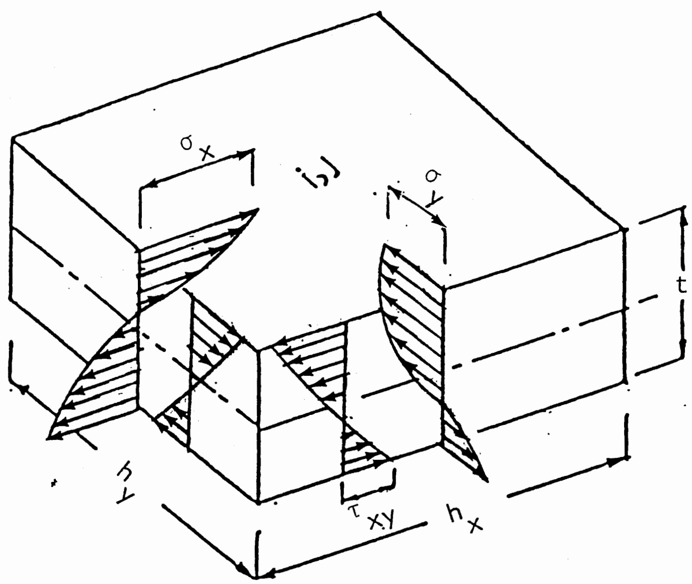
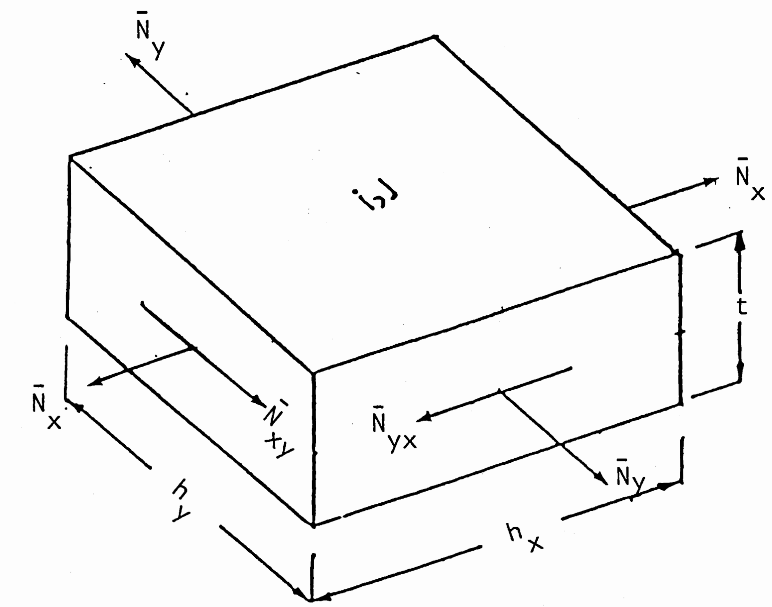
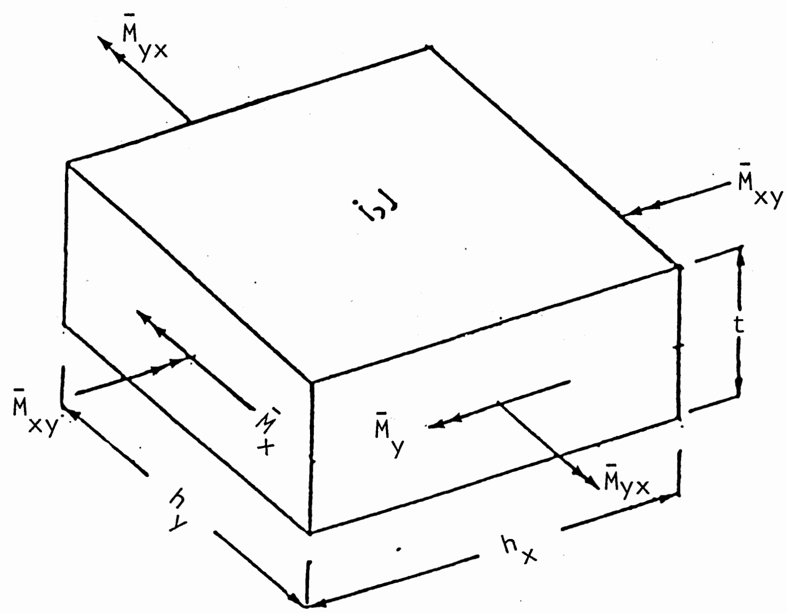


Figure 10. Stress Distribution at Node i,j



(a) Resultant Forces



(b) Resultant Moments

Figure 11. Stress Resultants at Node i,j

$((\bar{M}_x)_{i,j}$, $(\bar{M}_y)_{i,j}$, and $(\bar{M}_{xy})_{i,j})$ are related by means of equivalent orthotropic elastic bending constants $((E_{xb})_{i,j}$, $(E_{yb})_{i,j}$, $(G_{xyb})_{i,j}$, $(\nu_{xb})_{i,j}$, and $(\nu_{yb})_{i,j})$ as follows:

$$(\bar{M}_x)_{i,j} = \left[\frac{E_{xb}\ t^3}{12\ (1 - \nu_{xb}\ \nu_{yb})}\ (\phi_x + \nu_{yb}\ \phi_y)\ h_y\right]_{i,j} \tag{3.8a}$$

$$(\bar{M}_y)_{i,j} = \left[\frac{E_{yb}\ t^3}{12\ (1 - \nu_{xb}\ \nu_{yb})}\ (\phi_y + \nu_{xb}\ \phi_y)\ h_x\right]_{i,j} \tag{3.8b}$$

$$(\bar{M}_{xy})_{i,j} = \left[\frac{G_{xy}\ t^3}{6}\ \phi_{xy}\ h_y\right]_{i,j} \tag{3.8c}$$

$$[\nu_{xb}\ E_{yb}]_{i,j} = [\nu_{yb}\ E_{xb}]_{i,j} \tag{3.8d}$$

$$(G_{xyb})_{i,j} = \left[\frac{\sqrt{E_{xb}\ E_{yb}}}{2\ (1 + \sqrt{\nu_{xb}\ \nu_{yb}})}\right]_{i,j} \tag{3.8e}$$

Likewise, a set of equivalent elastic orthotropic material properties $((E_{xm})_{i,j}$, $(E_{ym})_{i,j}$, $(G_{xym})_{i,j}$, $(\nu_{xm})_{i,j}$, and $(\nu_{ym})_{i,j})$ relate the mid-plane strains and the resultant forces as follows:

$$(\bar{N}_x)_{i,j} = \left[\frac{E_{xm}}{(1 - \nu_{xm}\ \nu_{ym})}\ (\varepsilon_{xm} + \nu_{ym}\ \varepsilon_{ym})\ h_y\right]_{i,j} \tag{3.9a}$$

$$(\bar{N}_y)_{i,j} = \left[\frac{E_{ym}\ t}{(1 - \nu_{xm}\ \nu_{ym})}\ (\varepsilon_{ym} + \nu_{xm}\ \varepsilon_{xm})\ h_x\right]_{i,j} \tag{3.9b}$$

$$(\bar{N}_{xy})_{i,j} = [G_{xym}\ \varepsilon_{xym}\ t\ h_y]_{i,j} \tag{3.9c}$$

$$[\nu_{xm}\ E_{ym}]_{i,j} = [\nu_{ym}\ E_{xm}]_{i,j} \tag{3.9d}$$

$$(G_{xym})_{i,j} = \left[\frac{\sqrt{E_{xm} \, E_{ym}}}{2 \, (1 + \sqrt{\nu_{xm} \, \nu_{ym}})} \right]_{i,j} \tag{3.9e}$$

By utilizing Equations (3.8) and (3.9), the bending and membrane equivalent elastic constants at node i,j are calculated. More details about the concept of equivalent elastic constants are presented in Appendix A. In this appendix two types of problems are studied: the first illustrates the variation of equivalent elastic material properties of a plate lamina, and the second demonstrates the variation of plate equivalent material properties. The properties are related to bending and membrane behavior and simulate a plate segment subjected to changing curvatures and strains.

CHAPTER IV

SOLUTION OF NONLINEAR EQUATIONS

In Chapter II, plastic flow rules, and equations of equilibrium and compatibility for thin orthotropic plates subjected to large deflections are reviewed. In Chapter III, a description of the plate model, force-deformation relations, and the concept of orthotropic equivalent elastic material constants are discussed. The objective of this chapter is to describe the method by which all these items of information are put together to form a solution for the problem of large deflection of thin plates in the elastic-plastic range of material behavior. The proposed solution method is incremental and piecewise linear. The equilibrium, compatibility, and plastic flow equations are satisfied by an iteration process for each load increment.

4.1 Overview of Methodology

The behavior of the plate is represented by two structural models: bending and membrane models. The bending model resists only lateral forces while the membrane model resists both in-plane and, because of geometry, lateral forces. The total load is applied incrementally to the model in several loading steps.

To determine the deflected shape, two solution procedures are employed. The first utilizes alternating bending and membrane solutions to achieve deflection convergence and compatibility. This procedure is used in the regime where the bending model resists a major portion of the monotonically increasing lateral load and continues to resist additional lateral load as the solution proceeds. A portion of the incremental load is applied to the bending model and the transverse deflections are calculated. From the lateral deflections, membrane forces are calculated and in-plane displacements and the lateral membrane resistance are predicted. The solution proceeds with the evaluation of a new load to be applied to the bending model. The procedure for predicting the new load to be resisted in bending is discussed in section 4.2. The process is repeated until the point with the greatest deflection exhibits the same calculated deflections from one iteration to the next.

At a point in the iterative process, the magnitude of the predicted loads carried by the bending model decreases. For this regime of load, a solution procedure which utilizes the composite model is employed. Increments of load are applied to the composite model and vertical displacements of this model are calculated. These displacements are impressed on the membrane model and its lateral resistance is determined. The lateral resistance of the membrane is then utilized to update the composite model stiffness. This procedure is repeated until deflection convergence is achieved. In either formulation, the final displacements at each loading are used to calculate the equivalent elastic material constants to be used for the calculation of deflection for the succeeding load. A description of the equilibrium formulations of the models is given in this chapter. The details of the models are presented in Appendices B, C, and D.

In this study it is assumed that the loads are gradually applied in several steps. During the course of the application of the loads, the material properties of the models have the opportunity to change to satisfy prescribed elastic-plastic behavior. A resonable way to evaluate changes is to achieve deflection convergence at intermediate loads of monotonically increasing magnitude until the total load is applied. When deflection convergence is reached and plastic flow and compatibility equations for each intermediate load are satisfied, the equivalent elastic material constants are calculated. These constants are then used to determine the plate's secant stiffness for calculating deflection due to the application of the successive load increment.

4.2 Solution of Equations

In this section details of methods for calculating the deflected shape of the plate are presented. The formulation for the regime where bending and membrane models are cyclically applied is first discussed. The attention is then focused on the second solution procedure which uses the composite model.

4.2.1 Iterative Procedure for Bending Model

Initially, we seek to find the equilibrium deflected shape of the plate due to load:

$$\{F^1\} = \frac{1}{N}\{F_T\} \tag{4.1}$$

where

$\{F^1\}$ = initial load;

N = number of loading steps; and

$\{F_T\}$ = total load.

At this stage the material properties are assumed to be elastic. To start the solution the load $\{f_B^1\}_1$, which is one-tenth of $\{F^1\}$, is applied to the bending model. The superscript denotes the loading step number and the subscript indicates the iteration cycle number. Deflections of the bending model are obtained by using the equation

$$[K_B^1]\ \{w^1\}_1 = \{f_B^1\}_1 \tag{4.2}$$

where $[K_B^1]$ is the initial secant stiffness matrix of the bending model; and $\{w^1\}_1$ is the initial deflections of the bending model. The calculated deflections are then impressed on the membrane model and the in-plane displacement and forces are determined. These forces are used to calculate the lateral geometric stiffness of the model. The lateral resistance of this model is calculated by

$$[K_M^1]_1\ \{w^1\}_1 = \{R^1\}_1 \tag{4.3}$$

where $[K_M^1]$ is the lateral geometric stiffness of the membrane model for the first loading step; and $\{R^1\}_1$ is the transverse resistance of the membrane model. The total resistance of the structure (i.e., the sum of the bending and membrane resistances) is compared with the current load. Due to nonlinearity, the equilibrium is not satisfied and the load applied to the bending model must be modified, and the solution procedure is repeated.

The method for predicting the bending model load utilizes the procedure suggested by Fujino and Ohsaka (40). According to this method, the modified load at the ith iteration of the jth loading is

$$\{f_B^J\}_i = \frac{\alpha\{f_B^J\}_{i-1}}{\{f_B^J\}_{i-1} + \{R^J\}_{i-1}}\ [\{F^J\} - \{f_B^J\}_{i-1} - \{R^J\}_{i-1}] + \{f_B^J\}_{i-1} \tag{4.4}$$

where α is a constant and equal to 0.3 in this study. The loads calculated by Equation (4.4) are applied to the bending model, and vertical displacements are calculated. Impressing these displacements on the membrane model leads to the calculation of in-plane displacements, forces, and, finally, the lateral resistance of the membrane. This process is repeated until the maximum deflections for two successive iterations agree within a prescribed tolerance. At this point the displacement field is used to calculate the equivalent elastic constants which will be utilized for the calculation of the deflections due to the following load increment.

Equations (4.2) and (4.3) for the ith iteration cycle of the jth loading step can be written as

$$[K_B^J] \; \{w^J\}_i = \{f_B^J\}_i \tag{4.5}$$

$$[K_M^J] \; \{w^J\}_i = \{R^J\}_i \tag{4.6}$$

If the predicted $\{f_B^J\}_{i+1}$ is less than $\{f_B^J\}_i$, then the magnitude of $\{R^J\}_{i+1}$ is also less than $\{R^J\}_i$. For this condition the composite model is utilized for deflection calculation.

4.2.2 Iterative Procedure for Composite Model

The composite model is made up of bending and membrane components. The stiffness of this model includes both bending and membrane effects. In this solution procedure, an increment of load is applied to the model and the lateral displacements are calculated. These displacements are impressed on the nodes of the membrane model and the in-plane forces are determined. The in-plane forces are utilized to update the membrane effects of the composite model stiffness matrix while the bending properties are assumed to be unchanged. The same load increment is again applied to the composite model and the procedure is repeated until the point with the greatest deflection exhibits the same calculated deflections in two succeeding iterations. At this stage, the displacement field is used to determine the equivalent elastic properties of the model. These properties are used when the next load increment is applied to the composite model.

The equilibrium equation for the first iteration when the ℓth load increment is applied to the composite model is

$$[[K_M]_1 + [K_B^{\ell-1}]] \{w^{\ell}\}_1 = \{F^{\ell}\} \tag{4.7}$$

where $[K_M]_1$ and $[K_B^{\ell-1}]$ are the membrane and bending model stiffnesses, respectively. These stiffnesses are calculated by utilizing the converged displacement field due to increment load $\{F^{\ell-1}\}$; $\{F^{\ell}\}$ is the ℓth load increment; and $\{w^{\ell}\}_1$ is the corresponding vector of lateral displacements. At the i-th iteration the equilibrium equation is

$$[(K_M)_i + (K_B^{\ell-1})] \{w^{\ell}\}_i = \{F^{\ell}\} \tag{4.8}$$

where $(K_M)_i$ is the membrane stiffness that includes the effect of in-plane forces calculated at the i-ℓth iteration.

To accelerate the deflection convergence, the average of every two consecutive lateral displacements are used to calculate the in-plane forces and update the membrane stiffness matrix.

CHAPTER V

COMPUTER PROGRAM

In the present study, the numerical method developed by Leesavan (1) for solving the problem of large deflection of plates in the elastic range of material behavior is extended. The computer program presented in this work is based on his model and includes the plastic flow laws given in Chapter II. The program is written in FORTRAN IV and requires 2.6 million bytes of storage for analyzing a plate of 20x20 grid with 20 laminae at each node. Program versatility permits the analysis of a variety of plate geometry, loading, and plastic stress-strain behavior.

5.1 Summary of the Program

A general flow chart for a nonlinear analysis procedure is shown in Figure 12. An incremental solution is employed to solve the problem. The total load is applied to the plate in N steps. At the kth loading step, the load applied to the plate is

$$\{F^k\} = \frac{k}{N}\{F_T\} \tag{5.1}$$

To determine the distribution of this load to bending and membrane models, an iterative solution procedure is adapted. At the ith iteration cycle of the kth load $\{F^k\}$, the portion resisted by bending is $\{f_B^k\}_i$. Deflections $\{w^k\}_i$ are calculated by the following equation:

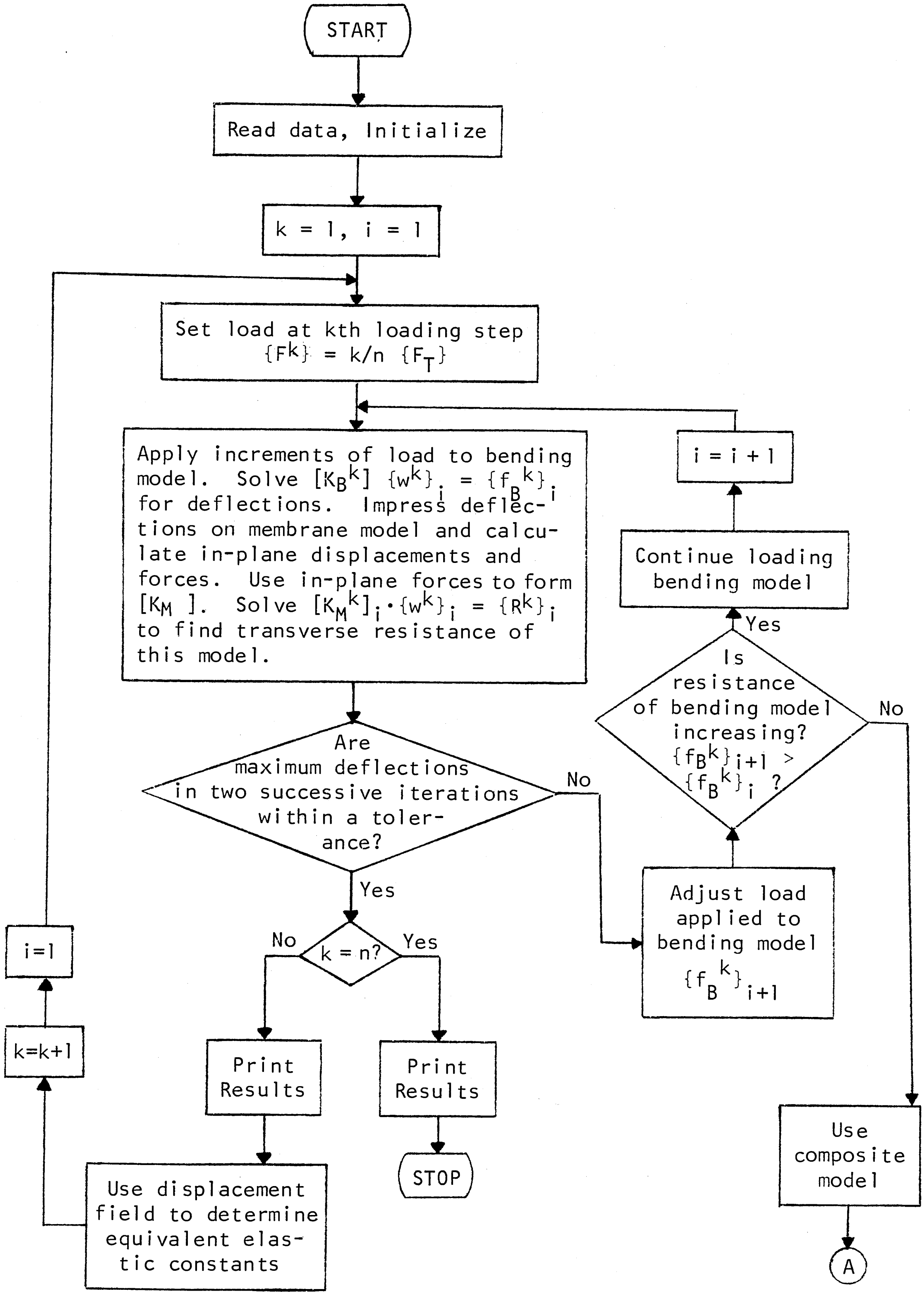


Figure 12. A Summary Flow Chart

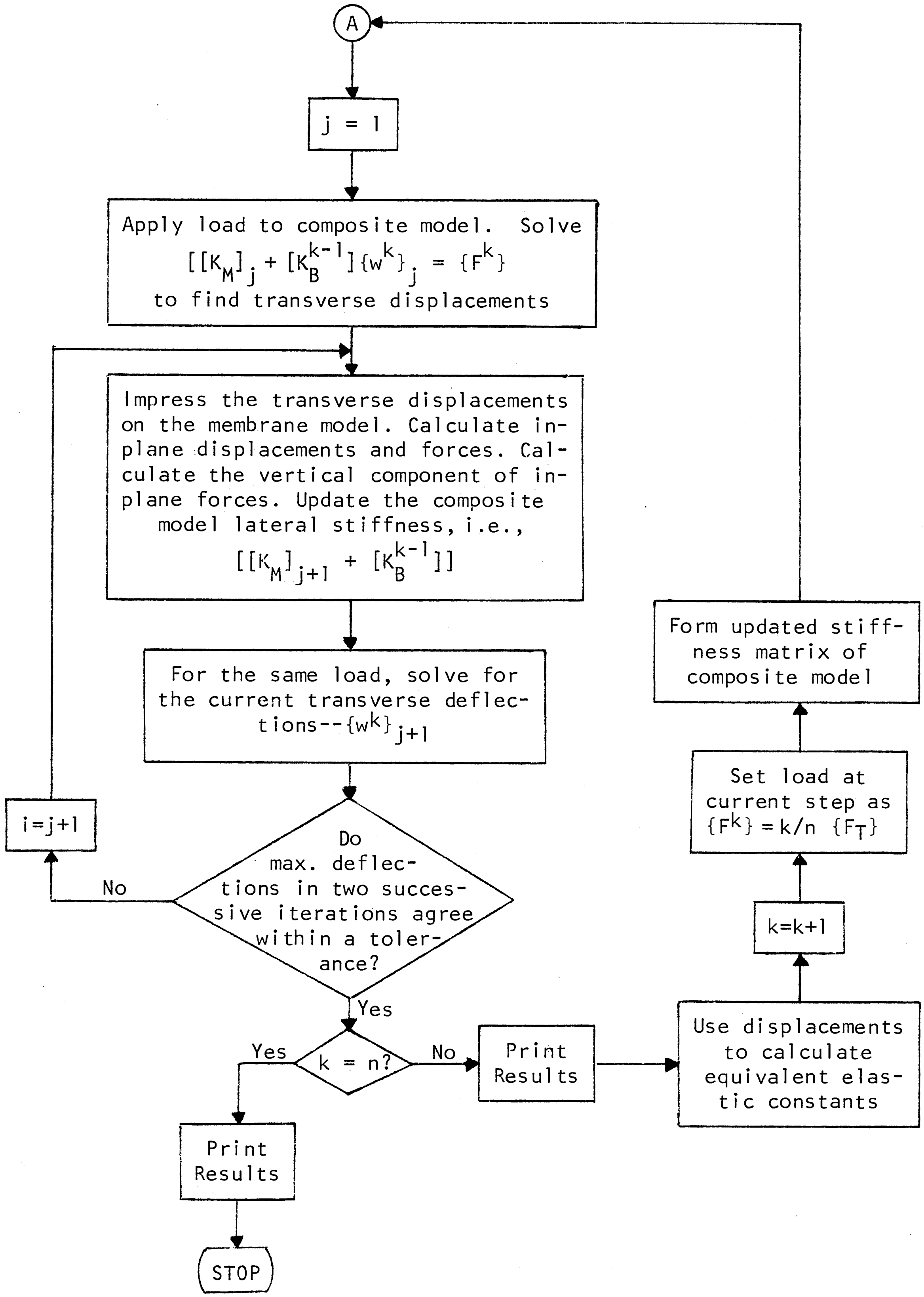


Figure 12. (Continued)

$$[K_B^k] \; \{w^k\}_i = \{f_B^k\}_i \tag{5.2}$$

where $[K_B^k]$ is defined in Appendix B. These deflections are impressed on the membrane model, and in-plane displacements and forces of this model are determined. These in-plane forces are used to form the lateral geometric stiffness $[K_M^k]$. The transverse resistance of the model is calculated by Equation (5.3) given below:

$$[K_M^k]_i \; \{w^k\}_i = \{R^k\}_i \tag{5.3}$$

If the maximum deflections for iterations i and i-1 are not within a given tolerance, an adjusted load vector $\{f_B^k\}_{i+1}$ is predicted and the above process is repeated until deflection convergence is achieved. At this stage displacements are used to evaluate the equivalent elastic constants to be used to form the bending and the membrane lateral stiffnesses used for the iteration process during the next step of loading.

If at any iteration cycle j of loading step k the predicted bending load $\{f_B^k\}_{j+1}$ happens to be less than $\{f_B^k\}_j$, a second solution procedure which utilizes the composite model is used. For this model lateral displacements are estimated by using Equation (4.7). The stiffness in this equation is the composite stiffness due to bending and geometry. The estimated deflections $\{w^k\}_1$ are impressed on the membrane model. Relevant in-plane displacements and forces are calculated and the vertical component of the in-plane forces are used to update the composite stiffness matrix (Appendix D). Transverse deflections for the same load are again calculated and convergence is reached when two consecutive deflection values agree within a stated tolerance. The displacements are used to find equivalent elastic constants. These constants are utilized to form the stiffness matrices to be used for iteration processes for load $\{F^{k+1}\}$. This iterative technique is repeated until the total load $\{F_T\}$ is applied to the model.

5.2 Overview of Input Data

In this section a summary of input information is presented. Output information is discussed in succeeding sections.

5.2.1 Description of Plate's Geometry, Boundary Conditions, and Material

Two alphanumeric cards are needed for describing the plate's geometry and boundary conditions followed by a single card. This card may contain problem number and a short statement about the plate's material.

5.2.2 Control Data (Table 1)

Three control cards are required for each problem. The first card specifies the number of cards in Tables 2, 3, and 4. The second card contains the number of increments into which the plate is divided in x and y directions (MX and MY), the length of these increments (HX and HY), Poisson's ratio for the elastic plate (PR), the thickness of the plate (THK), the maximum allowable number of iterations (NITER), and the deflection closure tolerance for the bending model (CLOS). THe third card contains the yield stress (YILSTZ), the yield strain (YSTAIN), the post-elastic tangent modulus (ALPHA), the number of loading steps (ISYM), and the deflection tolerance for the composite model (ALDF).

5.2.3 Plate Stiffness (Table 2)

The plate bending stiffness in the X and Y directions (DXN and DYN), twisting stiffness (CN), and the plate elastic moduli (EXN and EYN) are given in Table 2. The units of the bending and twisting stiffnesses are force/length^2. The maximum number of cards permitted in this table is 20.

5.2.4 Supporting Spring Stiffness (Table 3)

Vertical and in-plane spring stiffnesses (SN, SUN, SVN) are specified with a unit of force/length. The rotational spring stiffnesses (RXN and RYN) are of units of force-length/radians. Twenty cards are permitted in this table.

5.2.5 Load Data (Table 4)

The transverse applied force (QN) has units of force as do the applied in-plane forces (PXN and PYN). The applied couples (TXN and TYN) have force-length units. The maximum number of cards in this table is 20.

A blank card at the end of the data set is inserted to indicate the end of the run. There is no limit to the number of problems within a run.

5.3 Output Information

Both input data and calculated results are printed out in tabular form. Input information about the type of plate, its supports, and its material behavior are first printed. The rest of the input data are printed in Tables 1 through 4.

As mentioned before, the solution procedure in this study is incremental, i.e., the load $\{F_T\}$ is applied to the plate in several loading steps. The load applied during the kth step $\{F^k\}$ can be obtained from Equation (5.1). For each load step, the calculated plate's transverse and in-plane displacements are printed in Table 5. The number of iterations required to achieve deflection convergence is also printed in Table 5. If the maximum specified number of iterations are performed and deflection closure is not achieved, the closure error for the point of maximum deflection is printed in the same table. The bending and twisting moments of the nodes of the bending model are printed in Table 6, and the normal and shearing membrane stresses of the membrane model in Table 7. An example of the computer output is presented in Appendix F.

CHAPTER VI

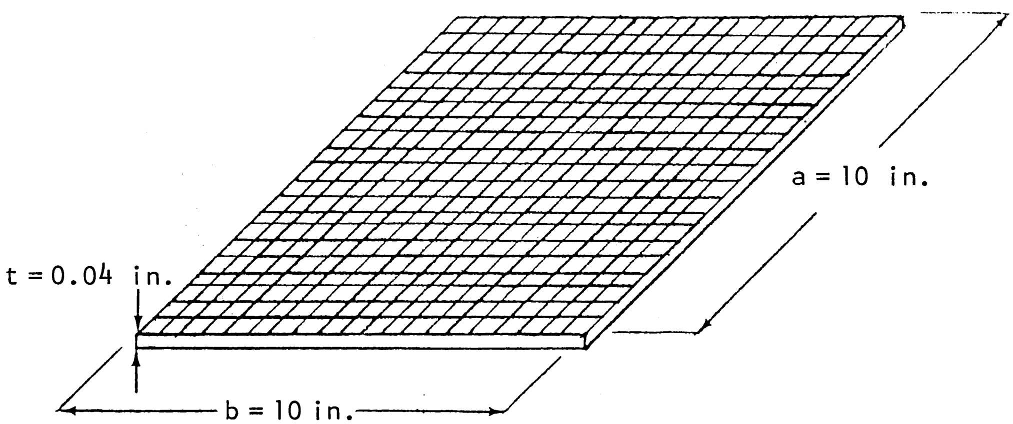
EXAMPLE PROBLEMS

In this chapter the results of the analysis of four example problems are presented. The example problems are the following: (1) elastic deflection of a simply supported plate with movable edges subjected to uniform lateral pressure; (2) elastic-plastic deflections of a simply supported plate with immovable edges subjected to uniform lateral pressure; (3) elastic-plastic bending of a plate strip subjected to central line load; and (4) elastic-plastic deflection of a rectangular plate with fixed edges subjected to lateral pressure. These four example problems demonstrate the method and are compared with the results, either analytical or experimental, reported by other investigators.

6.1 Elastic Behavior of a Square Plate With Vertical Edge Restraint

The first problem deals with large elastic deflections of a simply supported square plate subjected to uniform lateral pressure. The in-plane motion of the plate edges are unrestricted. The dimensions and elastic material properties of the plate are shown in Figure 13.

Results of analysis for this problem are shown in Figures 14, 15, and 16. In Figure 14 dimensionless central deflection of the plate, w_{max}/t, is plotted as a function of the dimensionless pressure, q_a^4/Et^4. Also shown in Figure 14 are the results given by Vallabhan (16). The



Simply supported square plate, movable edges subjected to uniformly distributed pressure, q = 0.307 psi

$h_x = h_y = 0.5$ in.

$\nu = 0.316$

$E_x = E_y = 10 \times 10^6$ lb/in.^{2}

$D_x = D_y = 59.25$ lb-in.

$C = 40.53$ lb-in.

$S = 1 \times 10^{50}$ lb/in.

$SU = SV = 1 \times 10^2$ lb/in.

Bending model deflection closure tolerance = 0.0005

Composite model deflection closure tolerance = 0.0002

Figure 13. Data for Elastic Large Deflection Analysis

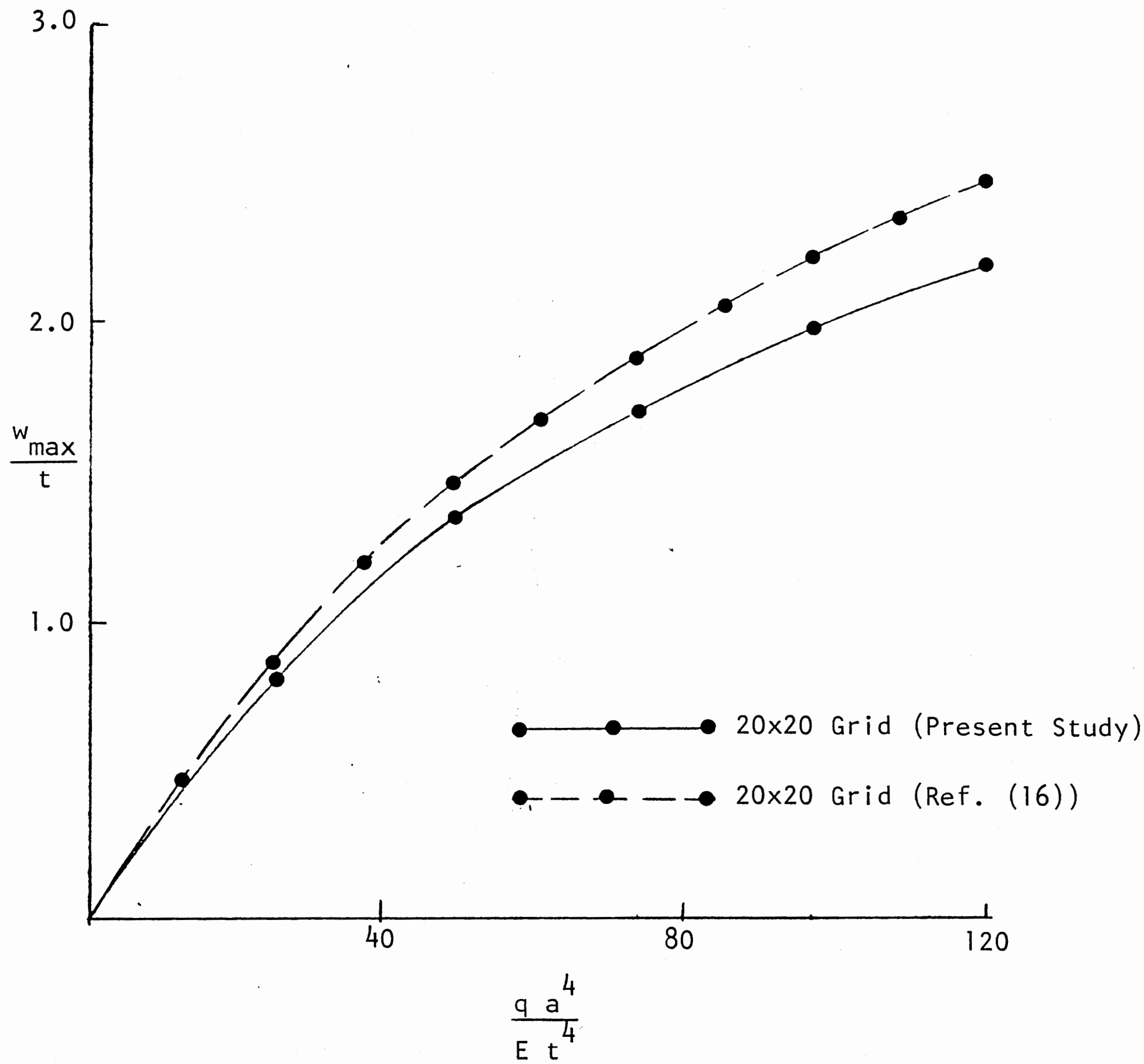


Figure 14. Center Point Deflection for Example Problem 1

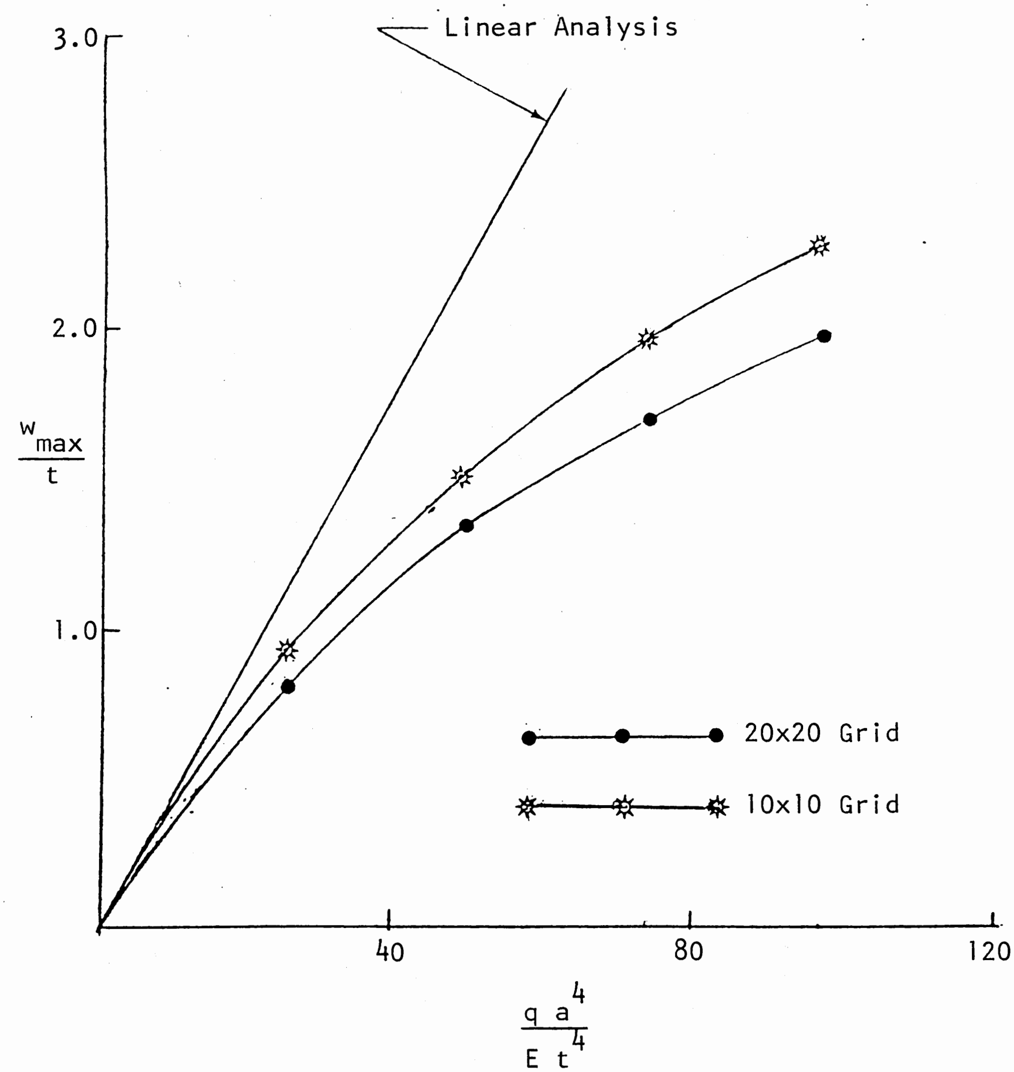


Figure 15. Study of the Effect of Mesh Size for Example Problem 1

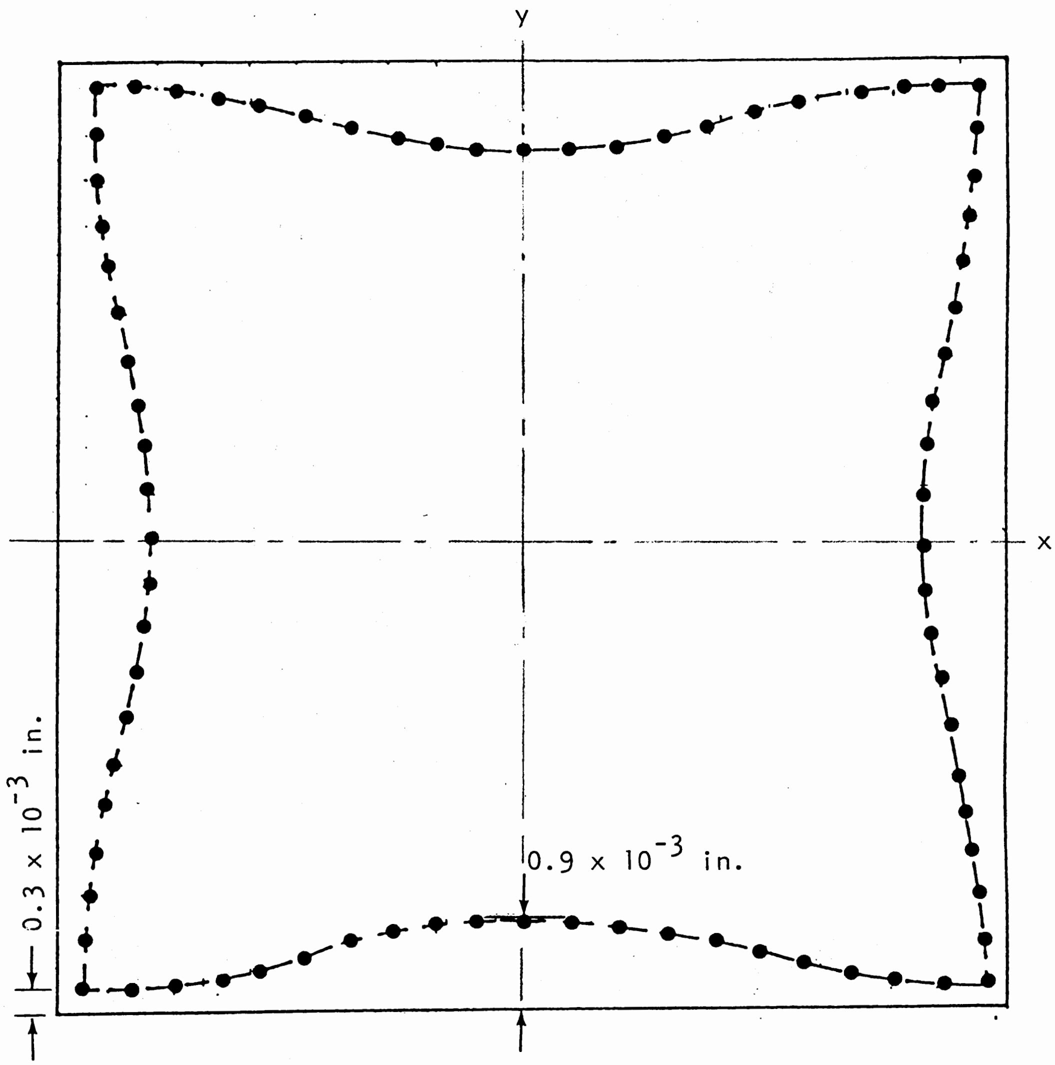


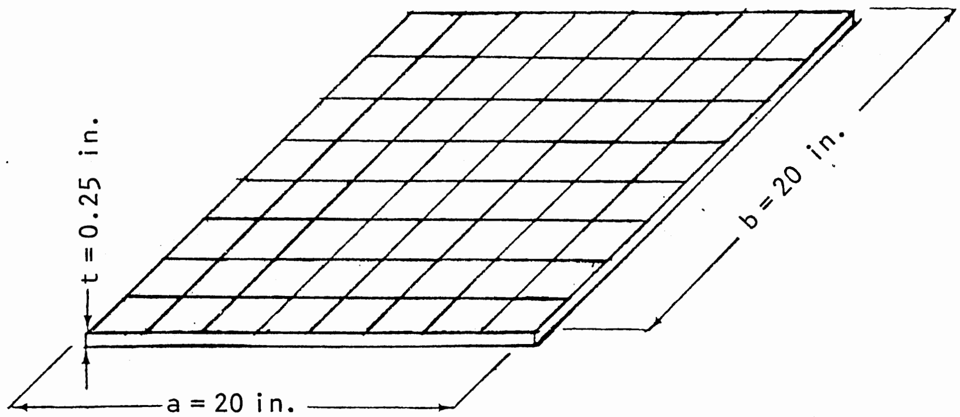
Figure 16. In-Plane Edge Displacements for $q = 0.307$ psi, Example Problem 1

results of this work compare favorably with those of Vallabhan's. However, the discrepancy between the magnitudes of the maximum deflections in both works may be due to the lateral springs used in the present study. (Vallabhan specifies free edges.) Although small in-plane restraining forces are applied to the edges, the forces may be sufficient to decrease the deflections shown in Figure 14. Figure 15 shows the variation of dimensionless maximum deflection versus dimensionless load for 10x10 and 20x20 grids. As the figure shows, the maximum deflection decreases as the grid size increases. Figure 16 shows the in-plane edge displacements due to a uniform lateral pressure, q = 0.307 psi. The displacements shown in this figure are exaggerated for clarity.

6.2 Elastic-Plastic Behavior of a Simply Supported Plate

This problem deals with the elastic-plastic large deflections of a square plate with edges fully restrained. The plate is loaded by a uniform, lateral pressure. The dimensions, elastic material properties, and stress-strain properties of the plate material are given in Figure 17.

For this problem, plots of the dimensionless maximum deflections versus dimensionless loads for 4x4 and 8x8 grids are given in Figure 18. The results obtained by the procedure proposed in this work are compared with those of Kuo-Kuang (25), whose analysis procedure is based upon a finite element model. General agreement of the results is evident from Figure 18. Shown in this figure is a consistent effect of grid size--the deflection decreases as the grid size increases. The variation of the moment along the centerline of the plate for uniform pressures--q = 7.33 psi, q = 14.66 psi, q = 65.90 psi, and q = 139.20 psi--is shown in Figure 19.



(a) Simply Supported Square Plate, Immovable Edges, Subjected to Uniformly Distributed Load, q = 146.50 psi

$h_x = h_y = 2.5$ in.

$\nu = 0.3$

$E_x = E_y = 10 \times 10^6$ lb/in.^{2}

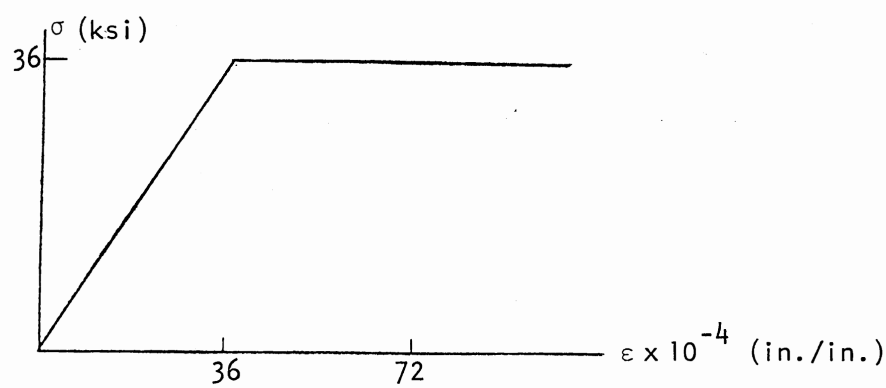
$D_x = D_y = 1.4309 \times 10^4$ lb-in.

$C = 1.0016 \times 10^4$ lb-in.

$S = SU = SV = 1.0 \times 10^{50}$ lb/in.

Bending model deflection closure tolerance = 0.0005

Composite model deflection closure tolerance = 0.0002



(b) Uniaxial Stress-Strain Relation

Figure 17. Data for Elastic-Plastic Large Deflection Analysis

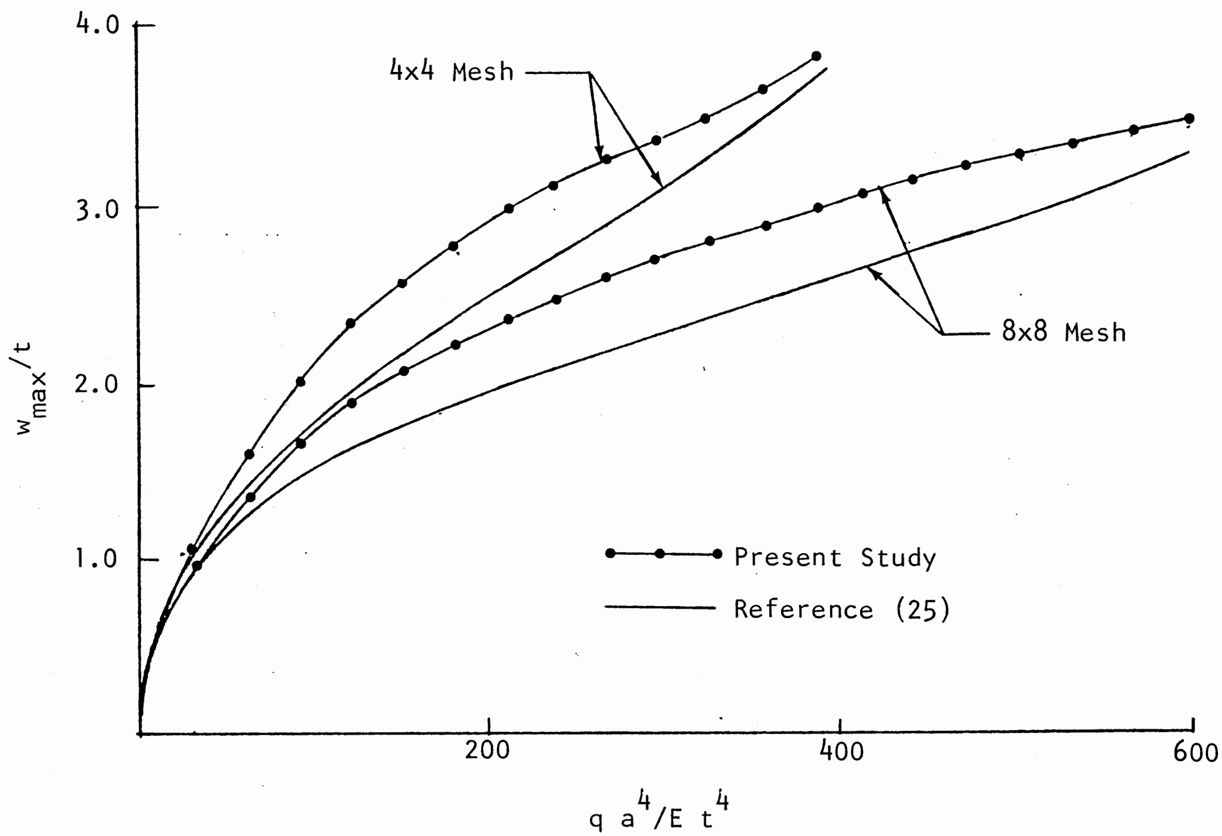


Figure 18. Center Point Deflection for Example Problem 2

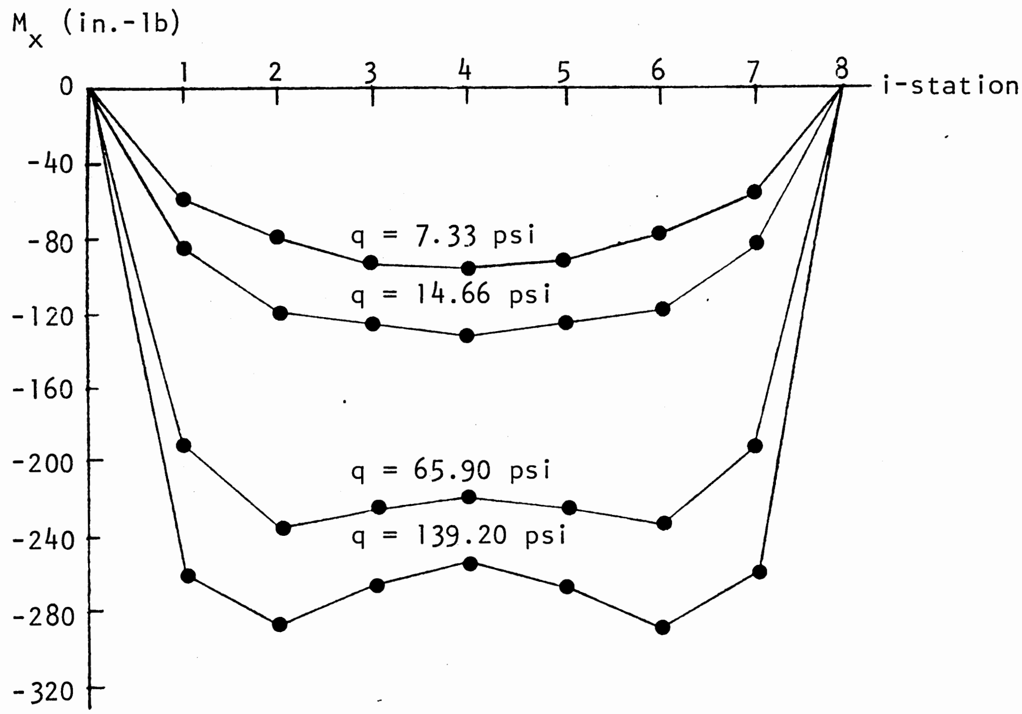


Figure 19. Moment for Elastic and Elastic-Plastic Behavior for Example Problem 2

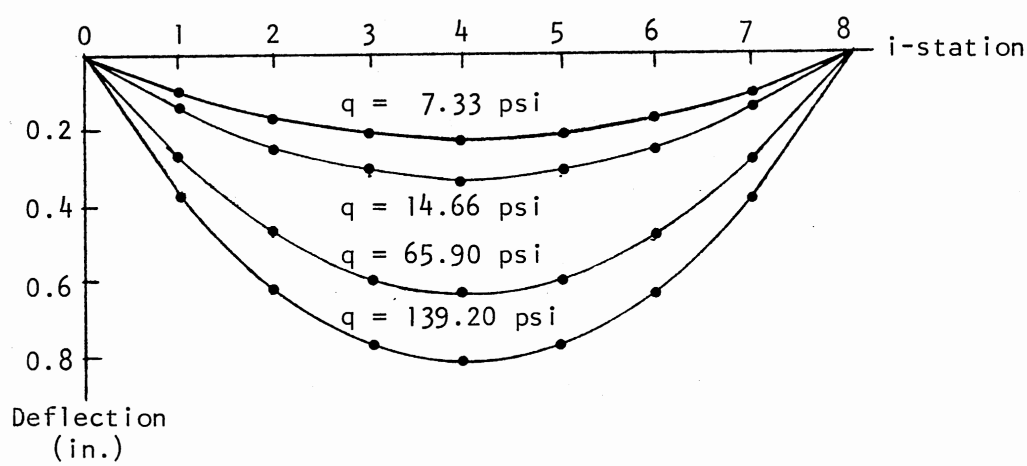


Figure 20. Transverse Displacements for Example Problem 2

Figure 20 shows the deflected shapes of the centerline of the plate when subjected to the above pressures. This figure illustrates the nonlinear load-deflection relation in the range of large deflections.

6.3 Approximate Elastic-Plastic Bending Analysis of a Two-Element Wide Model

A rectangular plate whose shorter edges are restrained from motion and longer edges are free to rotate and translate is subjected to a uniform line load at the centerline. The geometric and material properties of the plate are given in Figure 21. A grid of 2x10 is used for analysis of this plate. The free body diagram of half of the plate is shown in Figure 22. As seen in the figure, P is the support reaction; M and H are the moment and membrane force at the centerline of the plate. Plots of variations of P, H, and M versus maximum central deflection, w_o, are shown in Figure 22. The results of the approximate analysis by the method of this study are in general agreement with those obtained by Murray and Wilson (24). These authors use the finite element method to analyze the plate.

6.4 Elastic-Plastic Behavior of a Rectangular Plate With Fixed Supports

The fourth problem deals with the elastic-plastic large deflections of a rectangular plate with fixed edges subjected to uniform lateral pressure. The dimensions, elastic material properties, and the stress-strain plot for the material of this plate are shown in Figure 23. In Figure 24 the plot of dimensionless central deflection (w_o) versus lateral pressure obtained by the method discussed in this work is compared with the

(a) Rectangular Strip (14 x 2 x 0.5 in.) Simply Supported Along the Shorter Sides and Free Along the Long Edges Subjected to a Uniform Line Load Along the Centerline

$h_x = 1.4$ in., $h_y = 1.0$ in.

$\nu = 0.3$

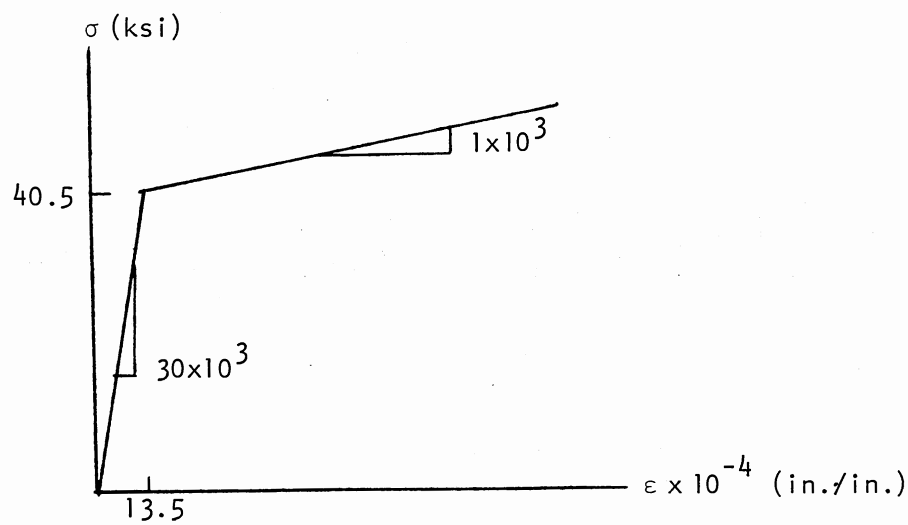
$E_x = E_y = 30 \times 10^6$ lb/in.^{2}

$D_x = D_y = 34.34 \times 10^4$ lb-in.

$C = 24.04 \times 10^4$ lb-in.

Bending model deflection closure tolerance = 0.0005

Composite model deflection closure tolerance = 0.0002



(b) Uniaxial Stress-Strain Relation

Figure 21. Data for Elastic-Plastic Large Deflection Analysis

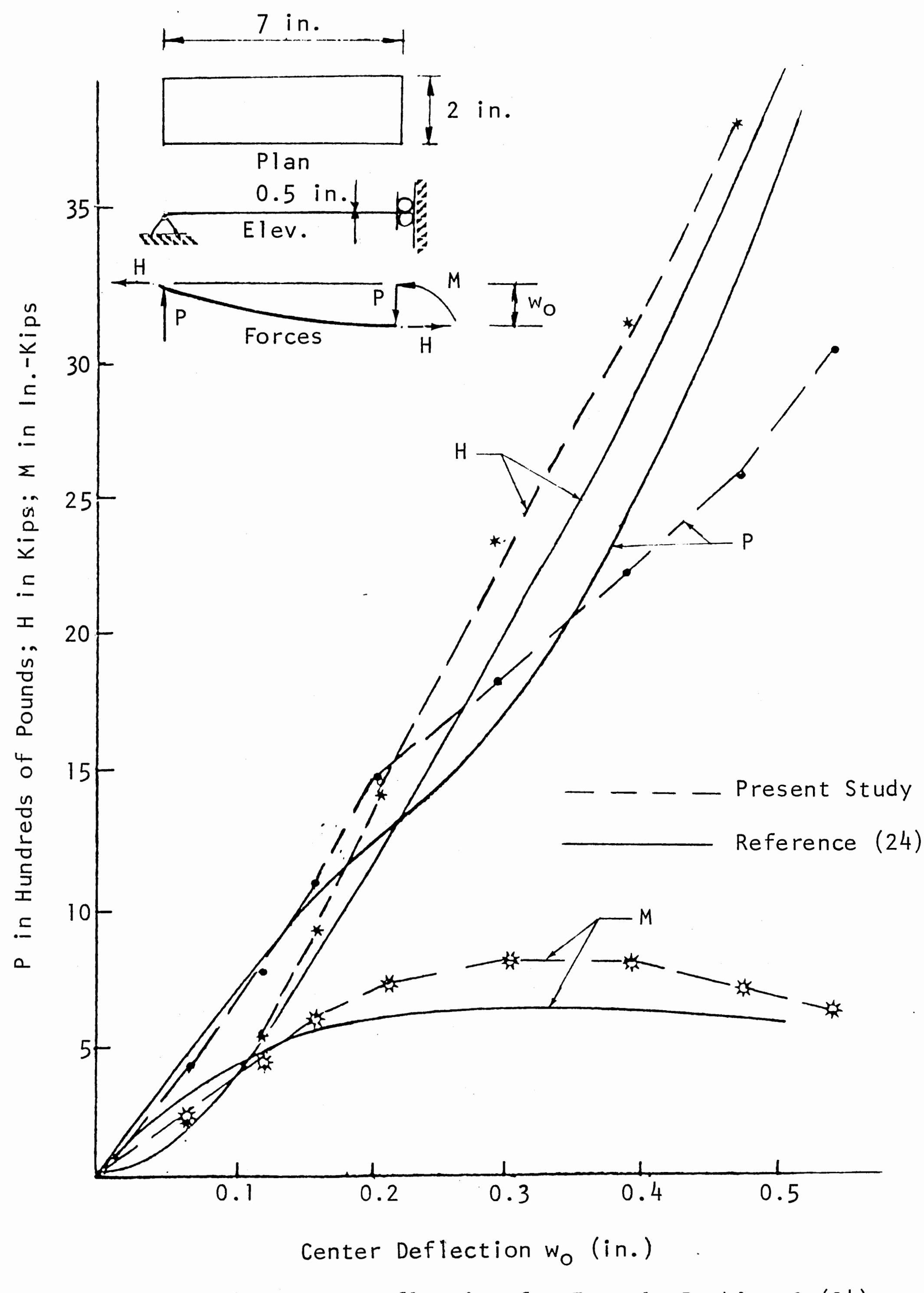
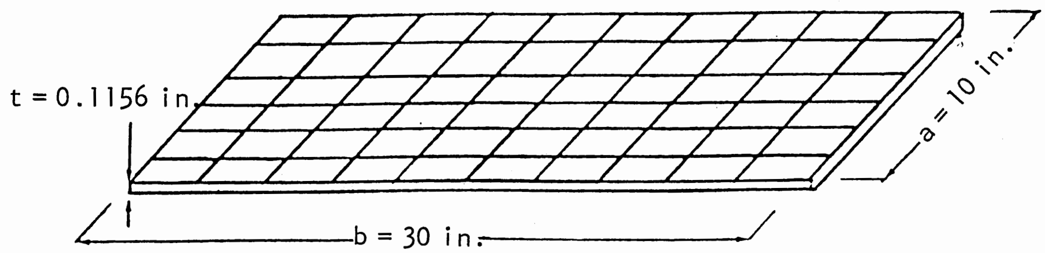


Figure 22. Force Deflection for Example Problem 3 (24)



(a) Rectangular Plate With Fixed Supported Subjected to Uniform Pressure, q = 50 psi

$h_x = 3.00$ in., $h_y = 1.67$ in.

$\nu = 0.3$

$E_x = E_y = 30 \times 10^6$ lb/in.^{2}

$D_x = D_y = 2.2440 \times 10^3$ lb-in.

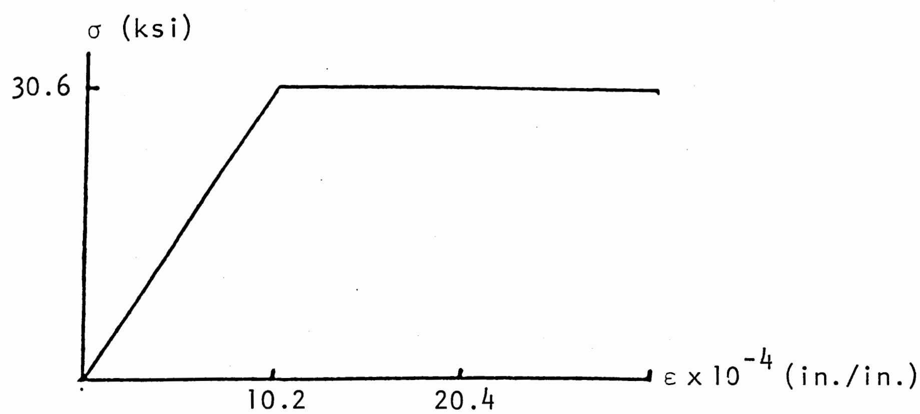
$C = 2.9708 \times 10^3$ lb-in.

$S = SU = SV = 1.0 \times 10^{50}$ lb/in.

$RX = RY = 1.0 \times 10^{10}$ lb-in./rad

Bending model deflection closure tolerance = 0.0005

Composite model deflection closure tolerance = 0.0002



(b) Uniaxial Stress-Strain Relation

Figure 23. Data for Elastic-Plastic Large Deflection Analysis

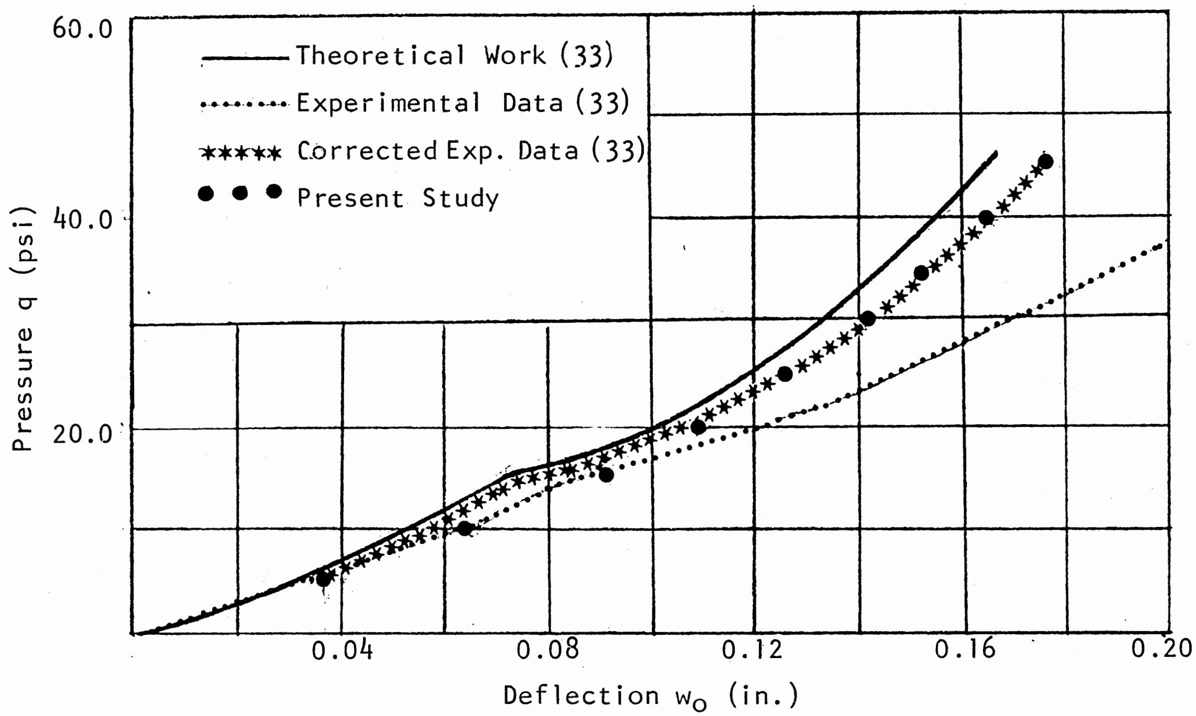


Figure 24. Center Deflections w_o (in.) for Example Problem 4 (33)

experimental data of Hooke and Rawlings (33). Correction to the experimental results was necessary because of deviations from perfect clamping. The analytical results out of the same reference are also included in Figure 24. An examination of this figure shows that the results obtained in this study compare favorably with the corrected experimental results of Reference (33).

CHAPTER VII

SUMMARY AND RECOMMENDATIONS

A summary of the analysis method and a review of the results are presented. Suggestions regarding the need for further research and use of the program are made.

7.1 Summary

The problem of large deflection of elastic-plastic plates involves both geometric and material nonlinearities. Closed-form solutions for this category of problems do not exist. Leesavan (1) proposes a numerical approach for solution of large deflection of thin plates in the elastic range. In the present study, Leesavan's approach is extended to include the material nonlinearity effects. The material properties are represented by an effective stress-effective strain generalization of a uniaxial tension test. The material model is linearly elastic-plastic.

The behavior of the plate is simulated by two structural models, one bending and one membrane. The bending model resists only lateral forces, while the membrane model resists both in-plane and, because of geometry, lateral forces. The total load is applied to the plate in several loading steps. At each loading step, deflection convergence is achieved and compatibility and stress-strain conditions are satisfied at all nodes by repeated application of the plate equilibrium equations.

Two methods are employed to determine the deflected shape of the plate. The first utilizes alternating bending and membrane solutions to achieve deflection convergence and compatibility. For any loading, deflections of the bending model are impressed on the membrane model and in-plane displacements and lateral membrane resistance are determined. Iteration proceeds with the prediction of the new load to be resisted by the bending model. The solution procedure is repeated until deflection convergence is achieved and compatibility is established. In this study convergence is achieved whenever the point with the greatest deflection exhibits the same calculated deflections, with a tolerance, in two successive iterations. If at a point in the iterative process the magnitude of the predicted loads for the bending model decreases, a modified iterative procedure is employed. The second procedure utilizes the composite bending and membrane model. For this analysis, loads are applied to the composite model and vertical displacements calculated. The displacements are impressed on the membrane model and bar forces, and lateral resistance determined. The new bar forces are used to revise the geometric stiffness of the model. Iteration is ended whenever the point with the greatest deflection exhibits the same calculated deflections, within a tolerance, at two successive iterations. When convergence is obtained for a load, the model displacements are used to calculate the equivalent elastic material constants to be used for the succeeding loading step.

Four plates are analyzed to demonstrate the versatility of the method. The first with elastic material properties has lateral edge supports and is lightly restrained against in-plane motion. The analysis of this plate compares favorably with analytical results obtained by

Vallabhan (16) who utilizes a nonlinear interpolation factor for accelerating the iterative solution of the Von Karman equations.

In the second example, a square plate with elastic-plastic material properties is restrained from transverse and in-plane displacement along all the edges. The results of analysis of this plate by the method of this study are in good agreement with the analytical results of Kuo-Kuang (25) who uses a finite element method to calculate the deflections.

For the third example, bending of a two-element wide model with elastic-plastic material properties is studied. The results of this approximate analysis are in general agreement with results obtained by Murray and Wilson (24) who utilize triangular finite elements to determine deflections and stress resultants.

For the final example, a plate made of elastic-plastic material with fixed edges is analyzed. The results of the analysis of this plate are in excellent agreement with the experimental data obtained by Hooke and Rawlings (33).

7.2 Recommendations

As demonstrated in Chapter VI, the model presented in this study can be utilized for analyzing plates subjected to various loading and boundary conditions. The structure of the model is useful for developing an understanding of the behavior of the plate. With regard to to these facts, it is recommended that the model be used for further investigation of plate behavior in the inelastic range of material properties.

The proposed solution method utilizes several iterative processes. The method of successive approximations, which can be a time-consuming procedure, is used to perform the iterations. Substitution of this procedure with a nonlinear interpolation technique may result in saving much computing time.

The solution procedure presented in this study utilizes a secant stiffness method and the concept of equivalent elastic constants in a Lagrangian Coordinate system. The total load is applied to the plate model in several steps. A study is recommended to evaluate the optimum number of load steps necessary for convergence to the equilibrium path. It is also recommended that a study be conducted to develop a solution based on tangent stiffness. Furthermore, the process for finding the resistance of the bending model should be reviewed. A nonlinear adjustment may prove to be suitable for the acceleration of deflection convergence.

While this study concentrates on plates subjected to monotonically increasing loads, it is recommended that the problem of unloading be addressed as an extension of this work. This extension would be necessary to encompass the dynamic response of plates.

The program may also be utilized to determine large deflections of elastic-plastic membranes and to analyze large deflections of multilayered composites. For the latter study, the program must be expanded so that the material properties of different layers are read-in as input data. The subroutines dealing with the evaluation of equivalent material constants should also be modified to handle the change of properties of different layers during the course of loading.

The results of analysis presented in this study are in excellent agreement with the available analytical results and limited test data. However, additional experimental data are needed for further evaluation of the method and possible revisions of the solution procedure.

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APPENDIX A

CORRELATION OF PLATE CURVATURES AND STRAINS WITH THE EQUIVALENT MATERIAL PROPERTIES

Before developing the computer program presented in Appendix G, a preliminary study of the different features of the program was performed. As a part of this study and based on theoretical discussion presented in Chapter III, two FORTRAN programs were developed for calculating the lamina and plate equivalent elastic constants for elastic-plastic material.

A.1 Equivalent Properties for Plane Stress Problem

Two thin laminae of elastic-plastic material are subjected to increasing strains. The objective is to calculate the variations of the equivalent elastic properties for these laminae. The equations of the theory of plasticity in Chapter II are used to calculate the stresses on each lamina. The equivalent elastic properties are constants that relate the given lamina strains to the calculated stresses.

For one of these laminae, the material properties of which are shown in Figure 25a, the state of strain is as follows:

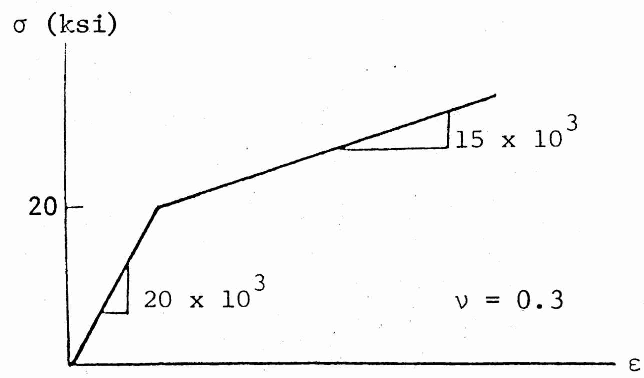
ε_x = increasing monotonically

$\varepsilon_y = 0.0004$

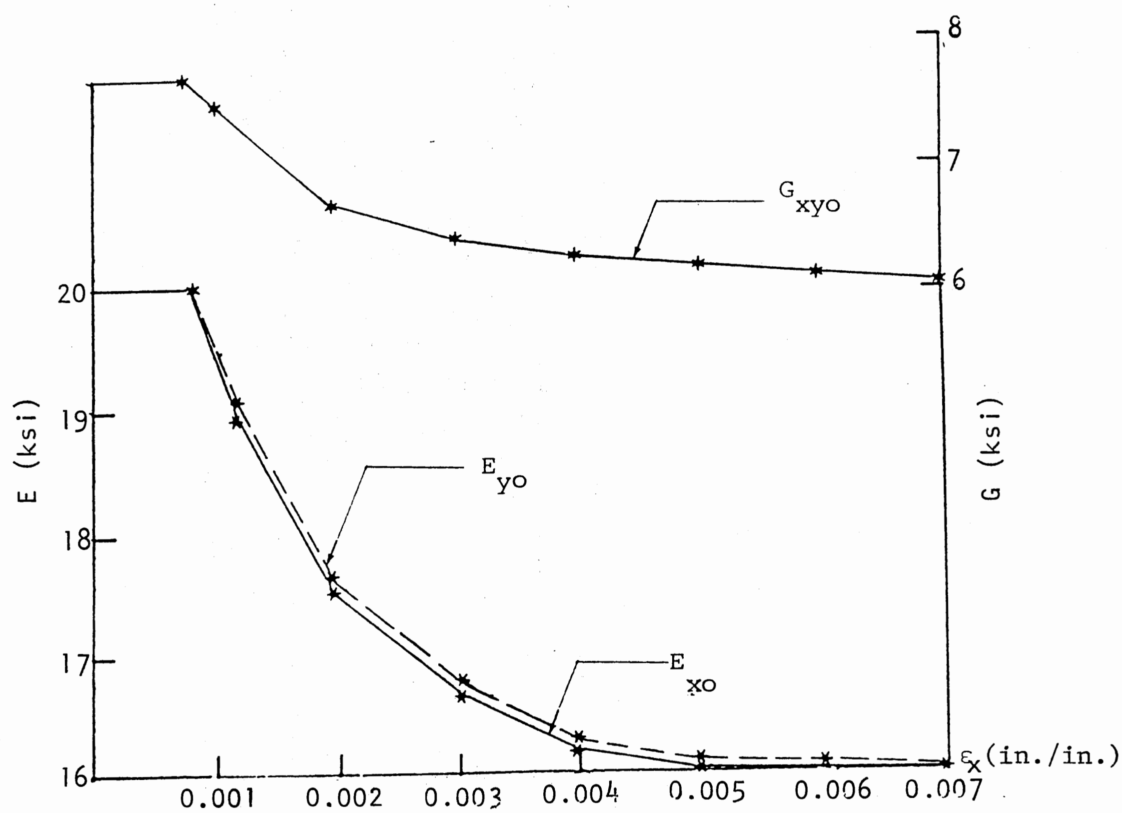
$\varepsilon_{xy} = 0.0002$

Variations of the equivalent orthotropic elastic constants (E_{xo}, E_{yo}, and G_{xyo}) versus ε_x (normal strain in the x direction) for this lamina are shown in Figure 25b. E_{xo} and E_{yo} are the equivalent orthotropic elastic moduli in the x and y directions; G_{xyo} is the equivalent elastic model of rigidity.

For the other lamina, the material properties of which are shown in Figure 26a, the state of strain is as follows:

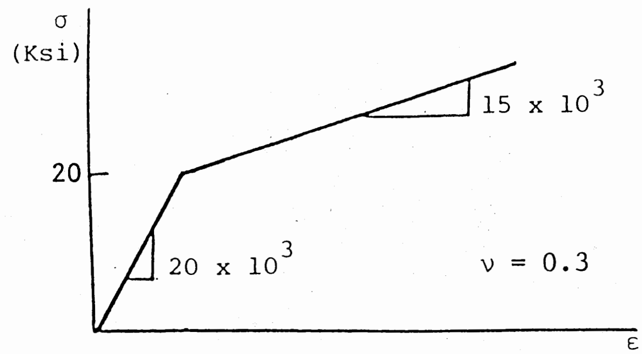


(a) Uniaxial Stress-Strain Relation

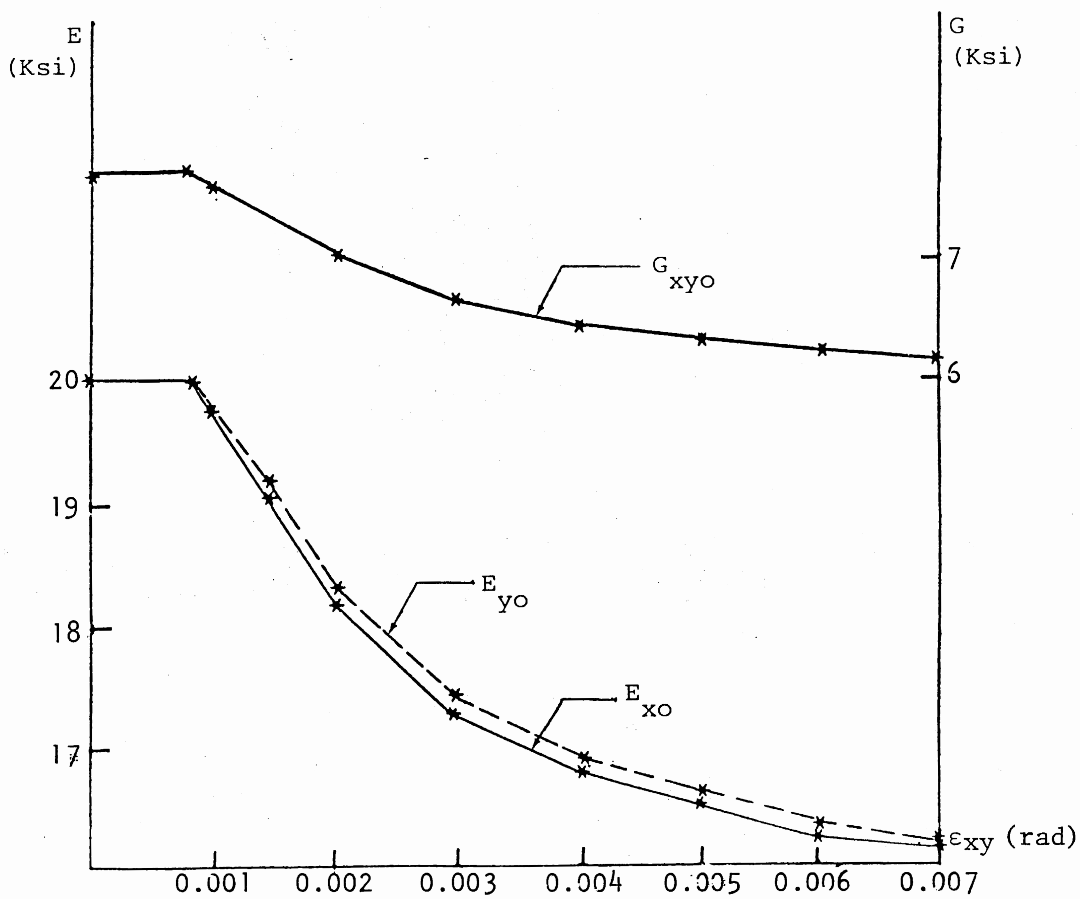


(b) Variation of Lamina Elastic Constants

Figure 25. Lamina Equivalent Elastic Moduli for Elastic-Plastic Material (Variation in Normal Strains)



(a) Uniaxial Stress-Strain Relation



(b) Variation of Lamina Elastic Constants

Figure 26. Lamina Equivalent Elastic Moduli for Elastic-Plastic Material (Variation in Shearing Strains)

$\varepsilon_x = 0.0007$

$\varepsilon_y = 0.0004$

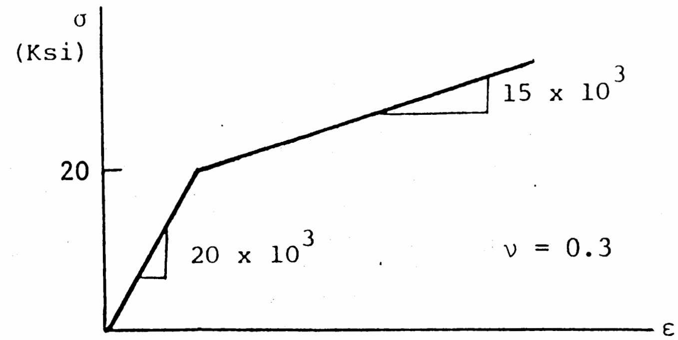
ε_{xy} = monotonically increasing

Plots of the variations of equivalent elastic material constants for this lamina versus ε_{xy} are shown in Figure 26b.

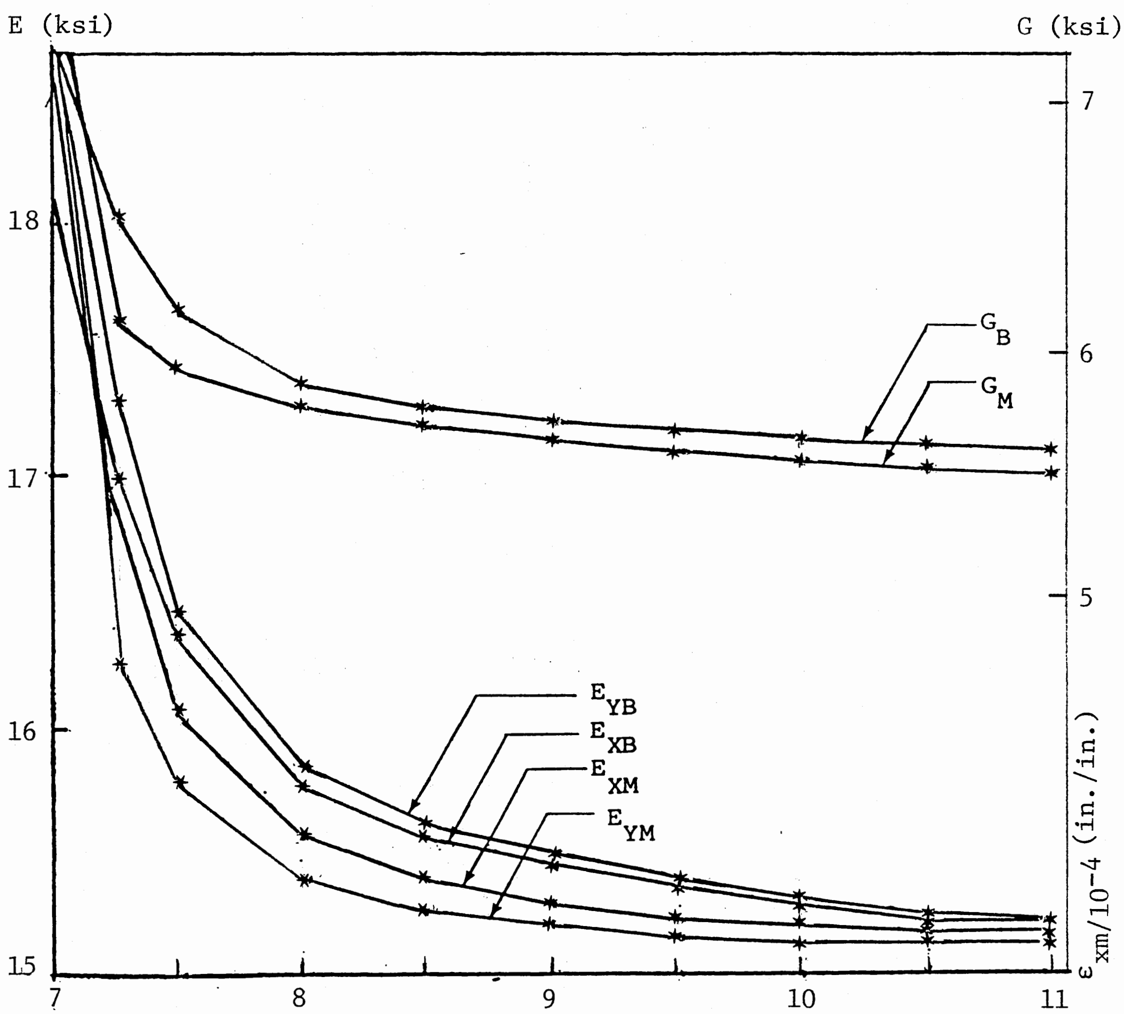
A.2 Equivalent Plate Properties

A plate of elastic-plastic material is subjected to a state of monotonically increasing curvatures (ϕ_x, ϕ_y, and ϕ_{xy}) and midsurface strains (ε_{xm}, ε_{ym}, and ε_{xym}). The objective is to evaluate the equivalent elastic bending and membrane material properties. For this purpose the plate is assumed to be made up of a number of frictionless laminae. Equation (3.5) given in Chapter III is utilized to calculate the state of strain in each plate lamina. The equations of the theory of plasticity given in Chapter II are used to calculate laminae stresses. The integration of the lamina stresses over the thickness of the plate yields stress resultants. Stress resultants, curvatures, and midsurface strains are related to equivalent orthotropic elastic constants by Equations (3.8) and (3.9).

As an example problem, a plate with material property shown in Figure 27a is subjected to a state of increasing curvatures and midsurface strains. This plate is assumed to be made of 100 laminae. By using the computer program developed for this study, it is possible to plot the variations of the plate's equivalent elastic bending constants (E_{xb}, E_{yb}, and G_{xyb}) and equivalent elastic membrane constants (E_{xm}, E_{ym}, and G_{xym}) versus changes in ε_{xm} (Figure 27b).



(a) Uniaxial Stress-Strain Relation



(b) Variation of Plate's Elastic Constants

Figure 27. Plate Equivalent Elastic Moduli for Elastic-Plastic Material (Variation in Curvatures and Midsurface Strains)

APPENDIX B

PROPERTIES OF THE BENDING MODEL

The bending model that simulates the bending action of the plate is shown in Figure 4. A detailed description of the makeup of this model and the functions of its different elements are given in Chapter III of this study and in References (1), (37), and (38). In this appendix, the equilibrium equation in the transverse direction for a joint of this model is presented. This equation is then extended to develop the stiffness of the bending model $[K_B]$.

B.1 The Equilibrium Equation for Bending Model

Figure 28 shows the free body diagram of a typical node i,j of the bending model. The equilibrium equation in the z direction for this node can be written as follows:

$$\Sigma F_Z = 0 = (f_B)_{i,j} + V^x_{i,j} + V^y_{i,j} - V^x_{i+1,j} - V^y_{i,j+1} - S_{i,j}\, w_{i,j} \tag{B.1}$$

The effects of rotational restraints are not included in Equation (B.1). The rotational restraint attached to node i,j in the x direction is represented in Figure 29. The rotational support at a joint resist rotation of the bars of the model by producing a moment proportional to the slope of the model at this joint. This resisting moment for node i,j of Figure 29 in the x direction is

$$CM^x_{i,j} = R^x_{i,j}\, (w_{i+1,j} - w_{i-1,j})/2h_x \tag{B.2}$$

where

$CM^x_{i,j}$ = couple developed at joint i,j;

$R^x_{i,j}$ = rotational stiffness at joint i,j in the x direction; and

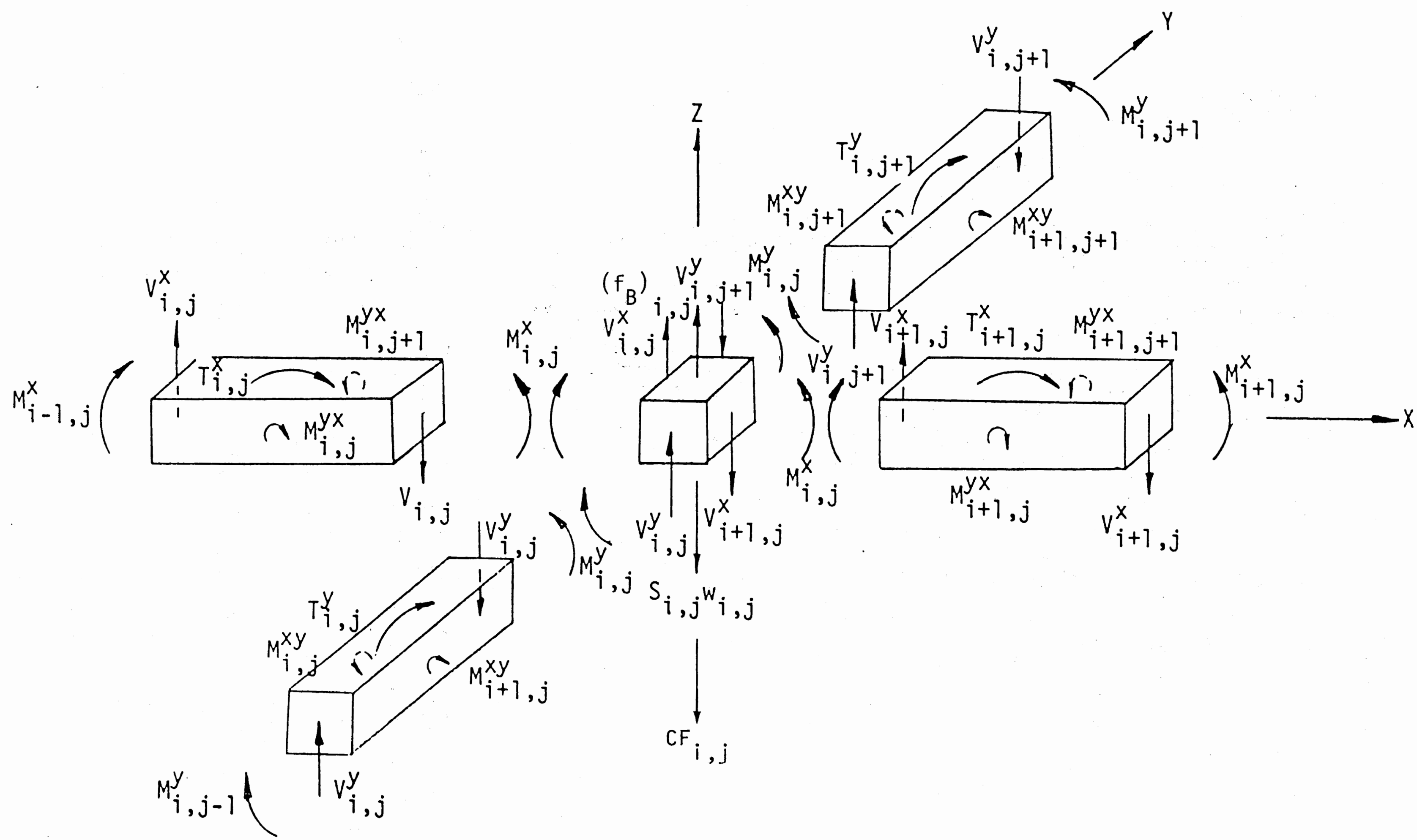


Figure 28. Expanded Joint of the Bending Model (1)

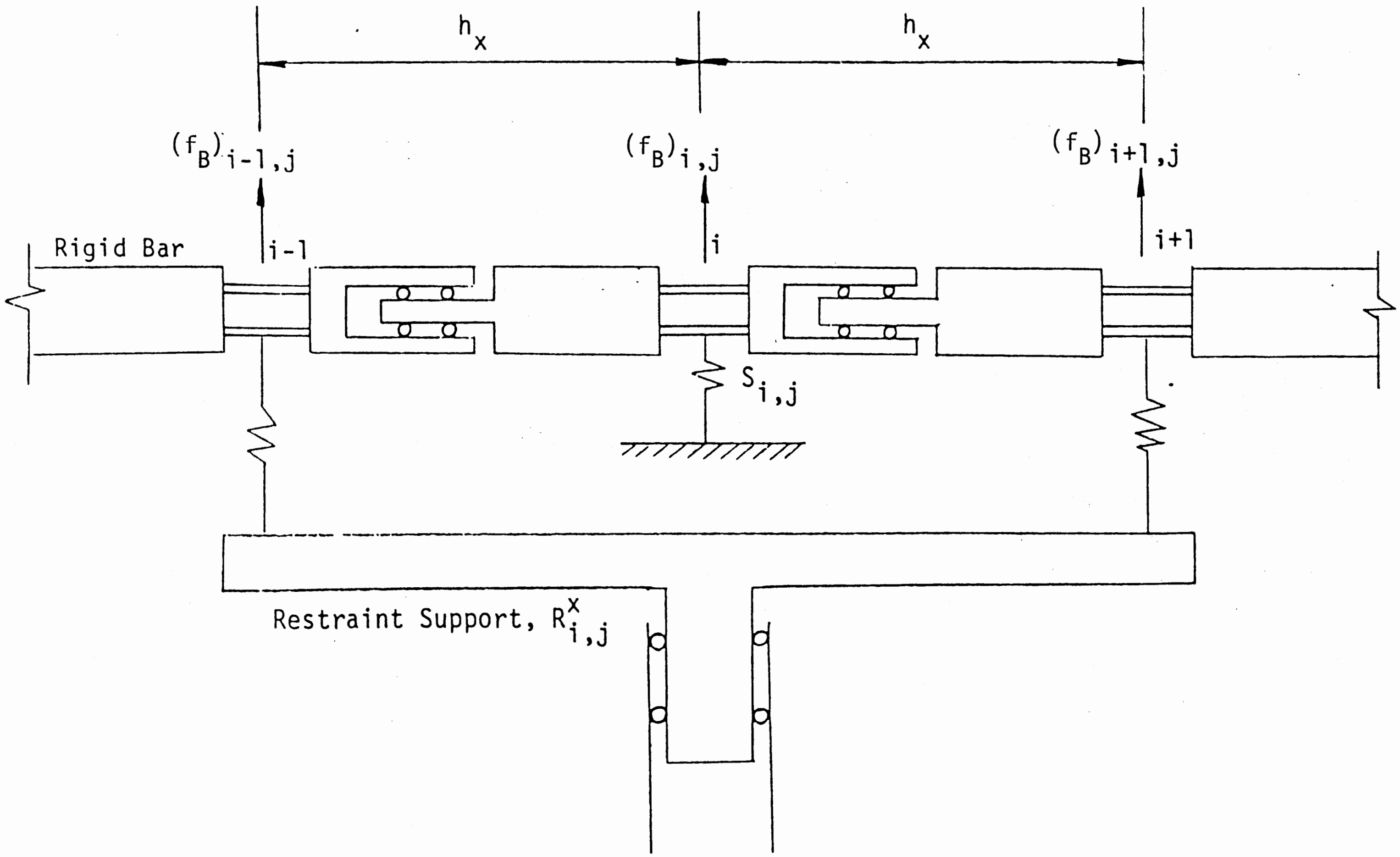


Figure 29. Typical Joint of Bending Model Showing Rotational Stiffness (1)

$(w_{i+1,j} - w_{i-1,j})/2h_x$ = slope of bars in the x direction at joint i,j.

Similarly, $CM^y_{i,j}$ developed in the y direction at node i,j can be expressed as follows:

$$CM^y_{i,j} = R^y_{i,j}\,(w_{i,j+1} - w_{i,j-1})/2h_y \tag{B.3}$$

where

$CM^y_{i,j}$ = couple developed at joint i,j;

$R^y_{i,j}$ = rotational stiffness at joint i,j in the y direction; and

$(w_{i,j+1} - w_{i,j-1})/2h_y$ = slope of bars in the y direction at joint i,j.

These couple moments can be expressed as couple forces applied to adjacent joints. The total force exerted at node i,j due to effects of rotational springs placed at the adjacent joints is

$$\begin{aligned} CF_{i,j} = {} & R^x_{i-1,j}\,(w_{i,j} - w_{i-2,j})/4h_x^2 \\ & + R^x_{i+1,j}\,(w_{i+2,j} - w_{i,j})/4h_x^2 \\ & + R^y_{i,j-1}\,(w_{i,j} - w_{i,j-2})/4h_y^2 \\ & + R^y_{i,j+1}\,(w_{i,j+2} - w_{i,j})/4h_y^2 \end{aligned} \tag{B.4}$$

Equation (B.1), modified to include the effect of rotational restraints, becomes

$$\begin{aligned} \Sigma F_z = 0 - (f_B)_{i,j} + V^x_{i,j} + V^y_{i,j} - V^x_{i+1,j} - V^y_{i,j+1} \\ - S_{i,j}w_{i,j} - CF_{i,j} \end{aligned} \tag{B.5}$$

Equation (B.5) can be used to generate the elements of the bending stiffness matrix, $[K_B]$. For this purpose the shearing forces and the term $CF_{i,j}$ in this equation must be expressed in terms of transverse joint displacements. Accordingly, the moment-equilibrium equations for the rigid bars connected to joint i,j are written as follows:

$$V^x_{i,j} = -(M^{yx}_{i,j} - M^{yx}_{i,j+1} + M^x_{i-1,j} - M^x_{i,j} + T^x_{i,j})/h_x \tag{B.6a}$$

$$V^x_{i+1,j} = -(M^{yx}_{i+1,j} - M^{yx}_{i+1,j+1} + M^x_{i,j} - M^x_{i+1,j} + T^x_{i+1,j})/h_x \tag{B.6b}$$

$$V^y_{i,j} = -(-M^{xy}_{i,j} + M^{xy}_{i+1,j} + M^y_{i,j-1} - M^y_{i,j} + T^y_{i,j})/h_y \tag{B.6c}$$

$$V^y_{i,j+1} = -(-M^{xy}_{i,j+1} + M^{xy}_{i+1,j+1} + M^y_{i,j} - M^y_{i,j+1} + T^y_{i,j+1})/h_y \tag{B.6d}$$

Substituting Equations (B.4) and (B.6) into Equation (B.5) yields

$$
\begin{aligned}
(f_B)_{i,j} - S_{i,j}w_{i,j} = \\
& - \frac{1}{4h_x^2} [R^x_{i-1,j}\, w_{i-2,j} - w_{i,j}\,(R^x_{i-1,j} + R^x_{i+1,j}) \\
& \qquad + R^x_{i+1,j}\, w_{i+2,j}] \\
& - \frac{1}{4h_y^2} [R^y_{i,j-1}\, w_{i,j-2} - w_{i,j}\,(R^y_{i,j-1} + R^y_{i,j+1}) \\
& \qquad + R^y_{i,j+1}\, w_{i,j+2}] \\
& + \frac{1}{h_x} [M^{yx}_{i,j} - M^{yx}_{i,j+1} - M^{yx}_{i+1,j} + M^{yx}_{i+1,j+1} + M^x_{i-1,j} \\
& \qquad - 2M^x_{i,j} + M^x_{i+1,j} + T^x_{i,j} - T^x_{i+1,j}]
\end{aligned}
$$

$$+ \frac{1}{h_y} [-M^{xy}_{i,j} + M^{xy}_{i+1,j} + M^{xy}_{i,j+1} - M^{xy}_{i+1,j+1} + M^{y}_{i,j-1}$$

$$- 2M^{y}_{i,j} + M^{y}_{i,j+1} + T^{y}_{i,j} - T^{y}_{i,j+1}] \qquad \text{(B.7)}$$

The bending and twisting moments in Equation (B.7) can be represented by finite difference approximations as

$$M^{x}_{i,j} = h_y D^{x}_{i,j} \left[\frac{w_{i-1,j} - 2w_{i,j} + w_{i+1,j}}{h_x^2} \right.$$

$$\left. + \nu_{yb} \frac{w_{i,j-1} - 2w_{i,j} + w_{i,j+1}}{h_y^2} \right] \qquad \text{(B.8a)}$$

$$M^{y}_{i,j} = h_x D^{y}_{i,j} \left[\frac{w_{i,j-1} - 2w_{i,j} + w_{i,j+1}}{h_y^2} \right.$$

$$\left. + \nu_{xb} \frac{w_{i-1,j} - 2w_{i,j} + w_{i+1,j}}{h_x^2} \right] \qquad \text{(B.8b)}$$

$$M^{xy}_{i,j} = -h_y C_{i,j} \left[\frac{w_{i-1,j-1} - w_{i-1,j} - w_{i,j-1} + w_{i,j}}{h_x h_y} \right] \qquad \text{(B.8c)}$$

$$M^{yx}_{i,j} = +h_x C_{i,j} \left[\frac{w_{i-1,j-1} - w_{i-1,j} - w_{i,j-1} + w_{i,j}}{h_x h_y} \right] \qquad \text{(B.8d)}$$

where

$$C_{i,j} = 2D^{xy}_{i,j}$$

The moment values of Equation (B.8) are substituted into Equation (B.7) and after rearranging the terms, the following equation is obtained:

$$a_{i,j}w_{i,j-2} + b_{i,j}w_{i-1,j-1} + c_{i,j}w_{i,j-1} + d_{i,j}w_{i+1,j-1}$$

$$+ e_{i,j}w_{i-2,j} + f_{i,j}w_{i-1,j} + g_{i,j}w_{i,j}$$

$$+ h_{i,j} w_{i+1,j} + p_{i,j} w_{i+2,j} + q_{i,j} w_{i-1,j+1}$$

$$+ r_{i,j} w_{i,j+1} + s_{i,j} w_{i+1,j+1} + t_{i,j} w_{i,j+2}$$

$$= u_{i,j} \tag{B.9}$$

where

$$a_{i,j} = \frac{h_x D^y_{i,j-1}}{h_y^3} - \frac{R^y_{i,j-1}}{4h_y^2}$$

$$b_{i,j} = \frac{(\nu_{yb} D^x_{i-1,j} + \nu_{xb} D^y_{i,j-1} + 2C_{i,j})}{h_x h_y}$$

$$c_{i,j} = \frac{-2h_x (D^y_{i,j-1} + D^y_{i,j})}{h_y^3}$$

$$- \frac{2(\nu_{yb} D^x_{i,j} + \nu_{xb} D^y_{i,j-1} + C_{i,j} + C_{i+1,j})}{h_x h_y}$$

$$d_{i,j} = \frac{(\nu_{yb} D^x_{i+1,j} + \nu_{xb} D^y_{i,j-1} + 2C_{i+1,j})}{h_x h_y}$$

$$e_{i,j} = \frac{h_y D^x_{i-1,j}}{h_x^3} - \frac{R^x_{i-1,j}}{4h_x^2}$$

$$f_{i,j} = \frac{-2h_y (D^x_{i-1,j} + D^x_{i,j})}{h_x^3}$$

$$\frac{-2(\nu_{yb} D^x_{i-1,j} + \nu_{xb} D^y_{i,j} + C_{i,j} + C_{i,j+1})}{h_x h_y}$$

$$g_{i,j} = \frac{h_y (D^x_{i-1,j} + 4D^x_{i,j} + D^x_{i+1,j})}{h_x^3}$$

$$+ \frac{h_x(D^y_{i,j-1} + 4D^y_{i,j} + D^y_{i,j+1})}{h_y^3}$$

$$+ \frac{(4\nu_{yb}D^x_{i,j} + 4\nu_{xb}D^y_{i,j} + 2C_{i,j} + 2C_{i+1,} + 2C_{i,j+1} + 2C_{i+1,j+1})}{4h_x^2}$$

$$+ \frac{(R^y_{i,j-1} + R^y_{i,j+1})}{4h_y^2} + S_{ij}$$

$$h_{i,j} = \frac{-2h_y(D^x_{i,j} + D^x_{i+1,j})}{h_x^3}$$

$$- \frac{2(\nu_{yb}D^x_{i+1,j} + \nu_{xb}D^y_{i,j} + C_{i+1,j} + C_{i+1,j+1})}{h_x h_y}$$

$$P_{i,j} = \frac{h_y D^x_{i+1,j}}{h_x^3} - \frac{R^x_{i+1,j}}{4h_x^2}$$

$$q_{i,j} = \frac{(\nu_{yb}D^x_{i-1,j} + \nu_{xb}D^y_{i,j+1} + 2C_{i,j+1})}{h_x h_y}$$

$$r_{i,j} = \frac{-2h_x(D^y_{i,j} + D^y_{i,j+1})}{h_y^3}$$

$$\frac{-2(\nu_{yb}D^x_{i,j} + \nu_{xb}D^y_{i,j+1} + C_{i,j+1} + C_{i+1,j+1})}{h_x h_y}$$

$$s_{i,j} = \frac{(\nu_{yb}D^x_{i+1,j} + \nu_{xb}D^y_{i,j+1} + 2C_{i+1,j+1})}{h_x h_y}$$

$$t_{i,j} = \frac{h_x D^y_{i,j+1}}{h_y^3} - \frac{R^y_{i,j+1}}{4h_y^2}$$

$$u_{i,j} = \frac{(f_B)_{i,j} - (T^x_{i,j} - T^x_{i+1,j})}{h_x} - \frac{(T^y_{i,j} - T^y_{i,j+1})}{h_y}$$

The above equilibrium equation is written for every node of the bending model. The resulting set of simultaneous equations can be symbolically expressed as

$$[K_B]\ \{w\} = \{f_B\} \tag{B.10}$$

where

$[K_B]$ = bending model stiffness matrix;

$\{w\}$ = lateral displacement vector; and

$\{f_B\}$ = lateral load vector applied to the bending model.

APPENDIX C

PROPERTIES OF THE MEMBRANE MODEL

The in-plane behavior of the plate is represented by the membrane model shown in Figure 2. This model is composed of elastic bars connected by ball and socket joints. The magnitude of the cross-sectional area of each bar is a function of the geometry of the plate and the in-plane material properties (1, 39).

The solution procedure in this study is incremental and stepwise linear. The total load is applied to the model in several loading steps. At each loading step, deflection convergence is achieved and compatibility conditions are satisfied. Prior to each loading, the bar cross-sectional areas of the membrane model are determined and remain constant during the iteration. In order to determine the cross-sectional areas of the bars, the plate is first subjected to in-plane normal and shearing stresses. The discrete element membrane model is also subjected to forces that are equivalent to these stresses. Equal deflections of both model and plate are established by the selection of suitable bar areas. The details of the procedure are explained in the following sections.

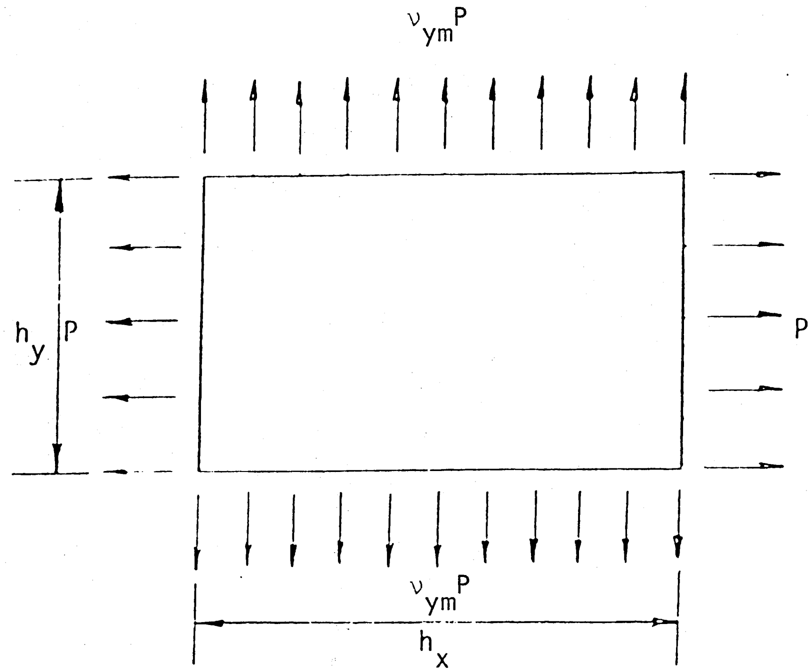
C.1 Bar Cross-Sectional Area

The plate shown in Figure 30a is subjected to a uniform normal in-plane load P per unit length in the x direction and ν_{ym} P in the y direction. The normal strains in the plate are

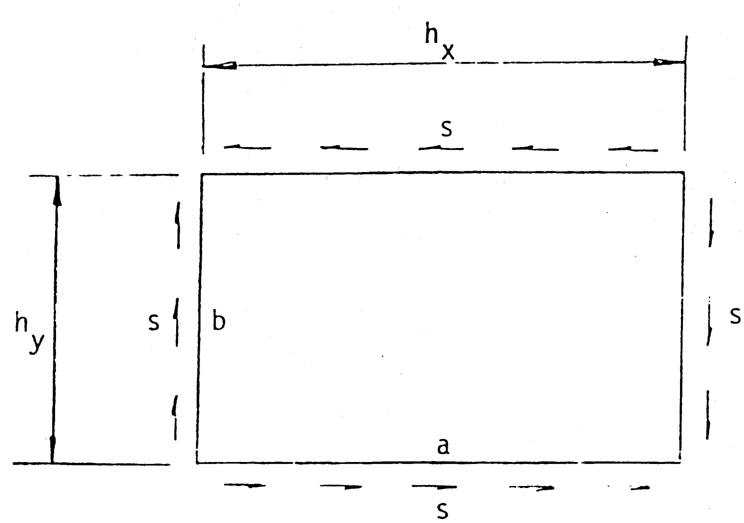
$$\varepsilon_x = \frac{P}{E_{xm}\, t}\,(1 - \nu_{xm}\,\nu_{ym}) \tag{C.1a}$$

$$\varepsilon_y = 0 \tag{C.1b}$$

If the same plate is subjected to a uniform in-plane load P per unit length in the y direction and ν_{xm} P in the x direction, the strains are



(a) Normal Loading



(b) Pure Shear

Figure 30. Plane Stress Element

$$\varepsilon_x = 0 \tag{C.2a}$$

$$\varepsilon_y = \frac{P}{E_{ym}\, t} (1 - \nu_{xm}\, \nu_{ym}) \tag{C.2b}$$

Finally, if the plate edges are subjected to a uniformly distributed in-plane shearing force s per unit length as shown in Figure 30b, the shearing strains are

$$\varepsilon_{xy} = \frac{s}{2G_{xy}\, t} \tag{C.3}$$

Figure 31a shows the discrete-element model that simulates the in-plane behavior of the plate. This model is made up of side bars a and b and diagonal bars c that are connected by ball and socket joints. Forces applied to the discrete-element joints in Figure 31a are statically equivalent to normal stresses P and ν_{ym} P applied to the plate in the x and y directions, respectively. Equilibrium equations for the joints of the model are

$$F_a + F_c \cos\theta = \frac{Ph_y}{2} \tag{C.4a}$$

and

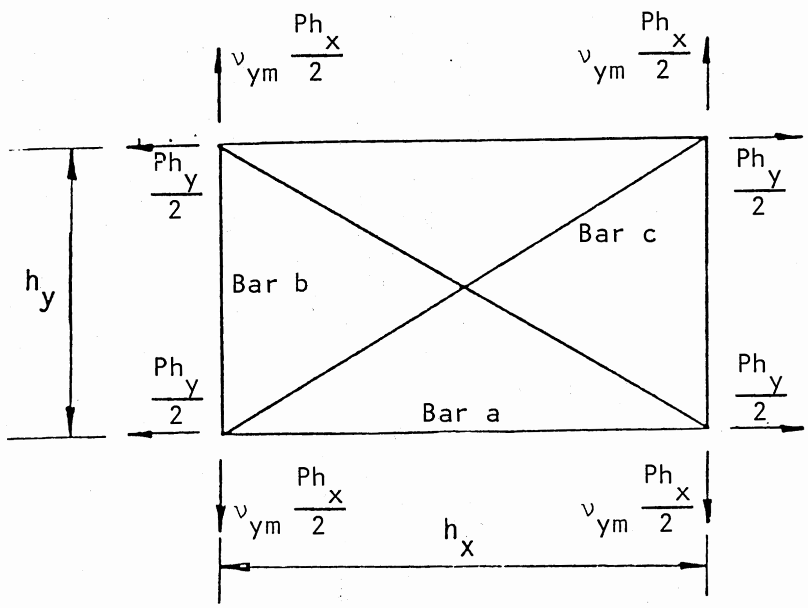
$$F_b + F_c \sin\theta = \frac{\nu_{ym}\, Ph_x}{2} \tag{C.4b}$$

where

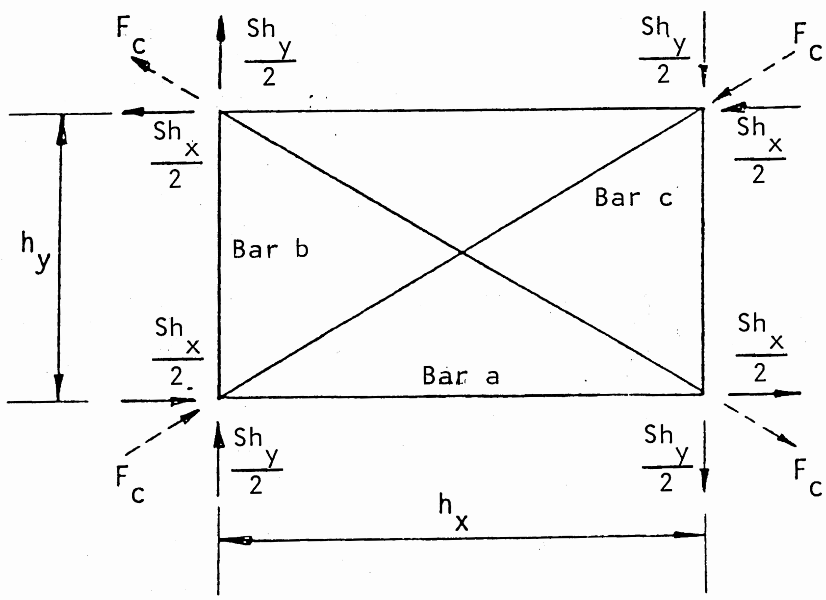
F_a, F_b, F_c = forces in bars a, b, and c, respectively;

θ = angle between bar a and bar c; and

h_x, h_y, h_z = lengths of bars a, b, and c, respectively.



(a) Normal Loading



(b) Pure Shear

Figure 31. Discrete-Element Model Equivalent Loads

The plate and model for this loading condition must have the same strains. The strain in the plate in the y direction is zero (Equation (C.1b)); consequently, the strain and the force in bar b of the model are zero, and Equation (C.4) can be rewritten as follows:

$$F_a = \frac{Ph_y}{2} - \frac{\nu_{ym} Ph_x^2}{2h_y} \qquad \text{(C.5a)}$$

$$F_c = \frac{\nu_{ym} Ph_x h_z}{2h_y} \qquad \text{(C.5b)}$$

The strain in bar a (ε_a) can now be written as

$$\varepsilon_a = \frac{F_a}{E_a A_a} = \frac{\frac{Ph_y}{2} - \frac{\nu_{ym} Ph_x^2}{2h_y}}{(E_{xm} A_a)} \qquad \text{(C.6)}$$

where A_a is the cross-sectional area of bar a. Equating Equations (C.1a) and (C.6) yields

$$A_a = \frac{t(h_y^2 - \nu_{ym} h_x^2)}{2h_y(1 - \nu_{xm}\nu_{ym})} \qquad \text{(C.7)}$$

Similarly, the cross-sectional area of bar b in the y direction is calculated as follows:

$$A_b = \frac{t(h_x^2 - \nu_{xm} h_y^2)}{2h_x(1 - \nu_{xm}\nu_{ym})} \qquad \text{(C.8)}$$

To determine the cross-sectional area of the diagonal bar c, the following equation is used:

$$h_y^2 = h_z^2 - h_x^2$$

Differentiating the above equation yields

$$h_y dh_y = h_z dh_z - h_x dh_x$$

Substituting $dh_x = \varepsilon_a h_x$, $dh_y = \varepsilon_b h_y$, and $dh_z = \varepsilon_c h_z$ into the above equation yields

$$h_y^2 \varepsilon_b = h_z^2 \varepsilon_c - h_x^2 \varepsilon_a$$

For the load condition at which the strain in bar b is zero, we can write

$$h_z^2 \varepsilon_c = h_x^2 \varepsilon_a$$

This equation can be rewritten in terms of the bar forces and the elastic moduli of bars a and c as follows:

$$\frac{h_z^2 \; F_c}{A_c \, E_c} = \frac{h_x^2 \; F_a}{A_a \; E_{xm}}$$

where A_c and E_c are the cross-sectional area and elastic modulus of bar c, respectively. Complete details for calculating E_c in terms of material properties of the x and y directions are given in Reference (41). The forces F_a and F_c were evaluated for this loading condition in Equation (C.5). By substituting for these forces from Equation (C.5) and rearranging terms, the area of bar c is found:

$$A_c = \frac{\nu_{ym} \; t \; h_z^3}{2h_x h_y (1 - \nu_{xm}\nu_{ym})} \cdot \frac{E_{xm}}{E_c} \tag{C.9}$$

The joint forces applied to the model in Figure 31b are the equivalent of the tangential load S per unit length applied to the plate element edges in Figure 30b. To satisfy the equivalence of the plate and the model, the forces in all horizontal and vertical members of the model must be zero. The magnitude of the force in each of the diagonal bars is as follows:

$$F_c = \frac{(Sh_x \cos\theta + Sh_y \sin\theta)}{2} \tag{C.10}$$

The change in length of each diagonal bar is

$$\delta = \frac{F_c h_z}{A_c E_c} = \frac{Sh_z^2}{2A_c E_c} \tag{C.11}$$

and the shearing strain with reference to Figure 32 is

$$\tan(w) = \frac{\delta \sin\theta}{h_x} = \frac{Sh_y h_z}{2h_x A_c E_c}$$

or, in other words

$$\varepsilon_{xy} = \frac{Sh_y h_z}{2h_x A_c E_c} \tag{C.12}$$

By equating the shearing strain of the plate as obtained in Equation (C.3) and the shearing strain of the model given by Equation (C.12) and by rearranging terms, we can obtain the cross-sectional area of bar c:

$$A_c = \frac{t\ G_{xy}\ h_y h_z}{h_x E_c} \tag{C.13}$$

For the model to be consistent, the magnitudes of the cross-sectional area of bar c obtained in Equations (C.9) and (C.13) have to be equal. This condition and the relation between elastic constants for the orthotropic material property of the plate are presented in Equations (C.14a) and (C.14b), respectively, as shown below:

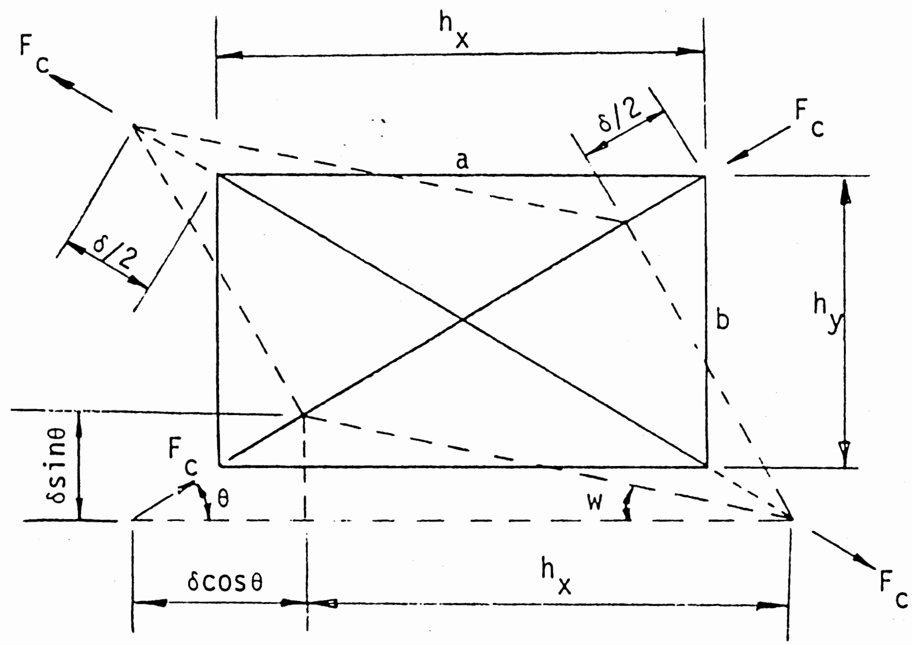


Figure 32. Shearing Deformation of Discrete Element (1)

$$\frac{G_{xy} h_y h_z t}{h_x E_c} = \frac{\nu_{ym} t E_{xm}}{E_c} \cdot \frac{h_z^3}{2h_x h_y (1 - \nu_{xm}\nu_{ym})} \tag{C.14a}$$

$$\nu_{xm} E_{ym} = \nu_{ym} E_{xm} \tag{C.14b}$$

Equations (C.14a) and (C.14b) can be combined in Equation (C.15) given below:

$$\frac{E_{xm}}{E_{ym}} (\nu_{ym})^2 + \frac{E_{xm} h_z^2}{2h_y^2 G_{xy}} (\nu_{ym}) - 1 = 0 \tag{C.15}$$

Therefore, the consistent Poisson ratio is a function of the geometry of the model and its elastic moduli. However, based on studies of Reference (1), the value of Poisson's ratio has little effect on the vertical displacements of the plate due to either vertical or in-plane loads.

C.2 Bar Forces Due to Lateral Deflections

When joints of the membrane model (Figure 4) undergo displacements, axial forces are produced in the bars. Figure 33 illustrates a diagonal member of the model before and after deformation. The original length of the diagonal is

$$h_z = (h_x^2 + h_y^2)^{1/2} \tag{C.16}$$

Following the deformation, the length of the diagonal changes to $\overline{h_z}$, where

$$\overline{h_z} = [(h_x + \Delta u)^2 + (h_y + \Delta v)^2 + \Delta w^2]^{1/2} \tag{C.17}$$

where, as seen in Figure 33,

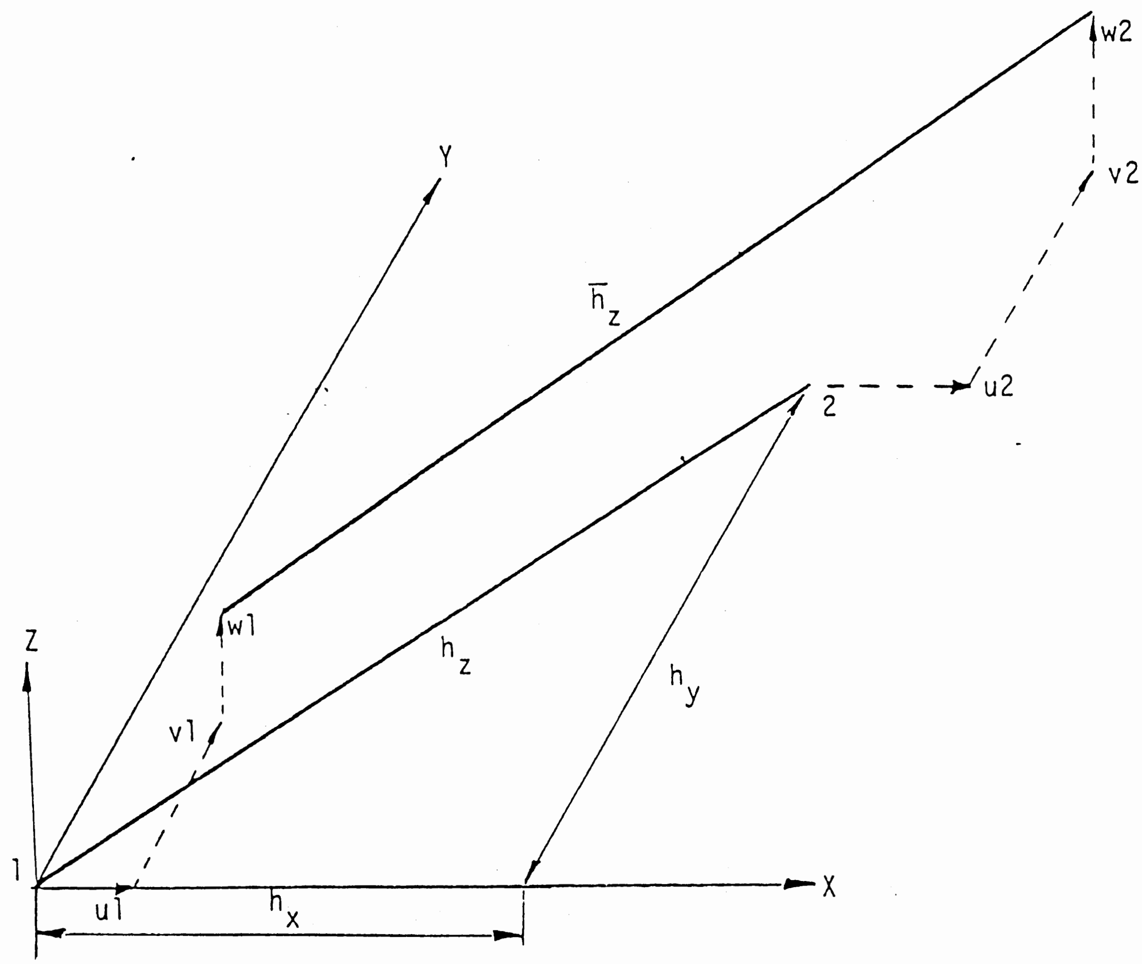


Figure 33. Deformation of an Elastic Bar of Discrete Element (1)

$\overline{h_z}$ = final length of the diagonal member;

$\Delta u = u_2 - u_1$;

$\Delta v = v_2 - v_1$; and

$\Delta w = w_2 - w_1$.

The axial strain in the deformed diagonal bar is

$$\varepsilon_c = \frac{(\overline{h_z} - h_z)}{h_z} \tag{C.18a}$$

By rearranging terms, this equation can be written as

$$\overline{h_z} = h_z + \varepsilon_c h_z \tag{C.18b}$$

By equating Equations (C.17) and (C.18b) and rearranging terms, we can find ε_c:

$$\varepsilon_c = \frac{h_x \Delta u + h_y \Delta v}{h_z^2} + \frac{(\Delta u)^2 + (\Delta v)^2 + (\Delta w)^2}{2h_z^2} \tag{C.18c}$$

The axial force in the deformed diagonal bar is

$$F_c = E_c A_c \left[\frac{h_x \Delta u + h_y \Delta v}{h_z^2} + \frac{(\Delta u)^2 + (\Delta v)^2 + (\Delta w)^2}{2h_z^2}\right] \tag{C.19}$$

This equation can be extended to find bar forces in members which lie in the x and y directions.

Equation (C.19) and its extensions can be utilized to calculate the approximate axial forces in bars of a membrane model produced by a set of transverse joint deflections. Since the sum of the x and y components of these forces at the joints is not zero, the joints are not at equilibrium. To restore equilibrium, the imbalancing joint forces are applied to the model and the in-plane joint displacements are calculated. Equation (C.19) is used again to find the actual bar force.

C.3 Model Geometric Stiffness

When the joints of the membrane model undergo transverse displacements, axial forces develop in the bars. The transverse components of these axial forces at each joint resist part of the load applied to the plate structure.

Figure 34 shows an expanded membrane model joint and the forces in adjoining bars. The vertical component of each force is equal to the magnitude of the force times the slope of the bar. The sum of these vertical components at joint i,j is termed $R_{i,j}$ and is as follows:

$$\begin{aligned} R_{i,j} = & -P^{x}_{i,j}\,(w_{i,j} - w_{i-1,j})/h_x + P^{x}_{i+1,j}\,(w_{i+1,j} - w_{i,j})/h_x \\ & - P^{y}_{i,j}(w_{i,j} - w_{i,j-1})/h_y + P^{y}_{i,j+1}\,(w_{i,j+1} - w_{i,j})/h_y \\ & - P^{c1}_{i,j}(w_{i,j} - w_{i-1,j+1})/h_z + P^{c1}_{i+1,j-1}(w_{i+1,j-1} - w_{i,j})/h_z \\ & - P^{c2}_{i,j}(w_{i,j} - w_{i-1,j-1})/h_z + P^{c2}_{i+1,j+1}(w_{i+1,j+1} - w_{i,j})/h_z \end{aligned} \tag{C.20}$$

When Equation (C.20) is written for all joints of the model, the resulting set of simultaneous equations may be symbolically expressed as

$$[K_M]\ \{w\} = \{R\} \tag{C.21}$$

where $[K_M]$ is the membrane model geometric stiffness; $\{w\}$ is the vector of transverse displacement; and $\{R\}$ is the vector of lateral membrane resistance.

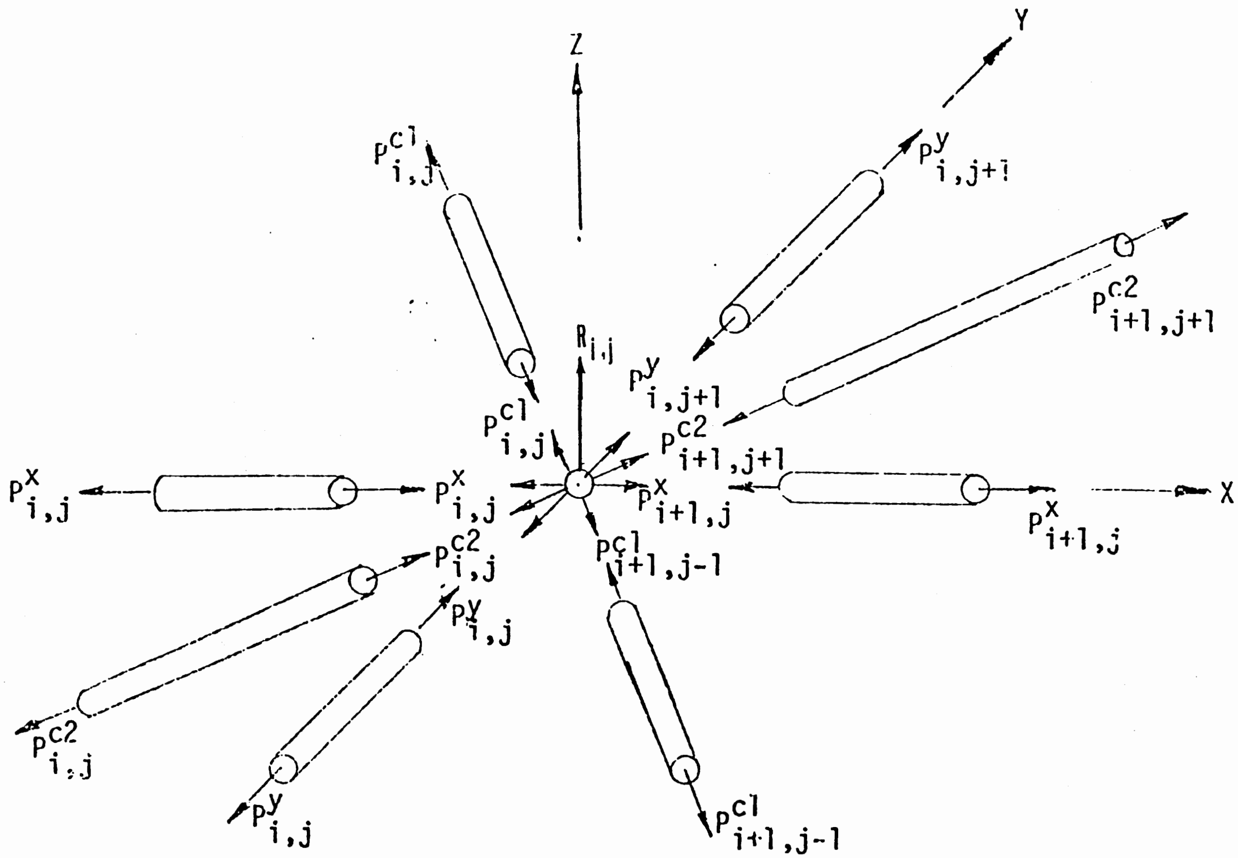


Figure 34. Expanded Joint of the Membrane Model (1)

C.4 Membrane Stress Evaluation

The bar forces determined by the method mentioned in section C.3 can now be used to evaluate the plate membrane stresses. The plate element shown in Figure 35 is subjected to in-plane forces indicated; normal and shearing stresses for this element of the plate are

$$\sigma_x = \frac{F_x}{th_y} \tag{C.22a}$$

$$\sigma_y = \frac{F_y}{th_x} \tag{C.22b}$$

$$\tau_{xy} = \frac{V_{xy}}{th_y} \tag{C.22c}$$

$$\tau_{yx} = \frac{V_{yx}}{th_x} \tag{C.22d}$$

The model that represents this plate element as shown in Figure 36 is subjected to joint forces which are equivalent to forces applied to the plate element. If F_A, F_B, F_C, F_D, F_E and F_H be forces in members of the model as shown in Figure 37, the equilibrium equations of the joints of the model in the x and y directions can be written as follows:

$$[F_B + F_H \cos\theta] = [F_x + V_{yx}]/2 \tag{C.23a}$$

$$[F_C + F_H \sin\theta] = [F_y + V_{xy}]/2 \tag{C.23b}$$

$$[F_B + F_E \cos\theta] = [F_x - V_{yx}]/2 \tag{C.23c}$$

$$[F_D + F_E \sin\theta] = [F_y - V_{xy}]/2 \tag{C.23d}$$

$$[F_A + F_E \cos\theta] = [F_x - V_{yx}]/2 \tag{C.23e}$$

$$[F_C + F_E \sin\theta] = [F_y - V_{xy}]/2 \tag{C.23f}$$

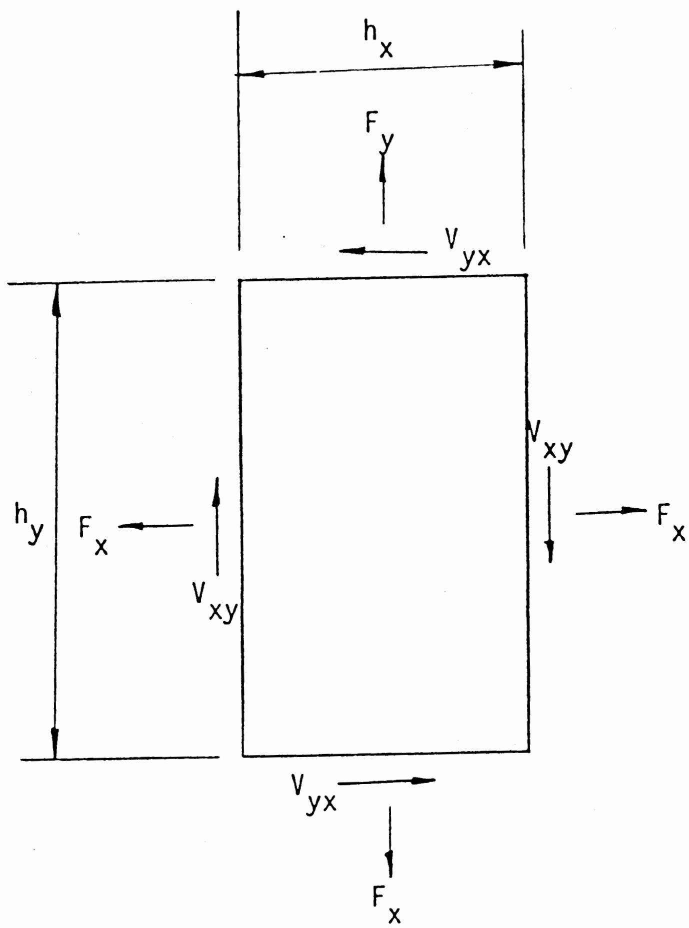


Figure 35. Plate Element With In-Plane Loads (1)

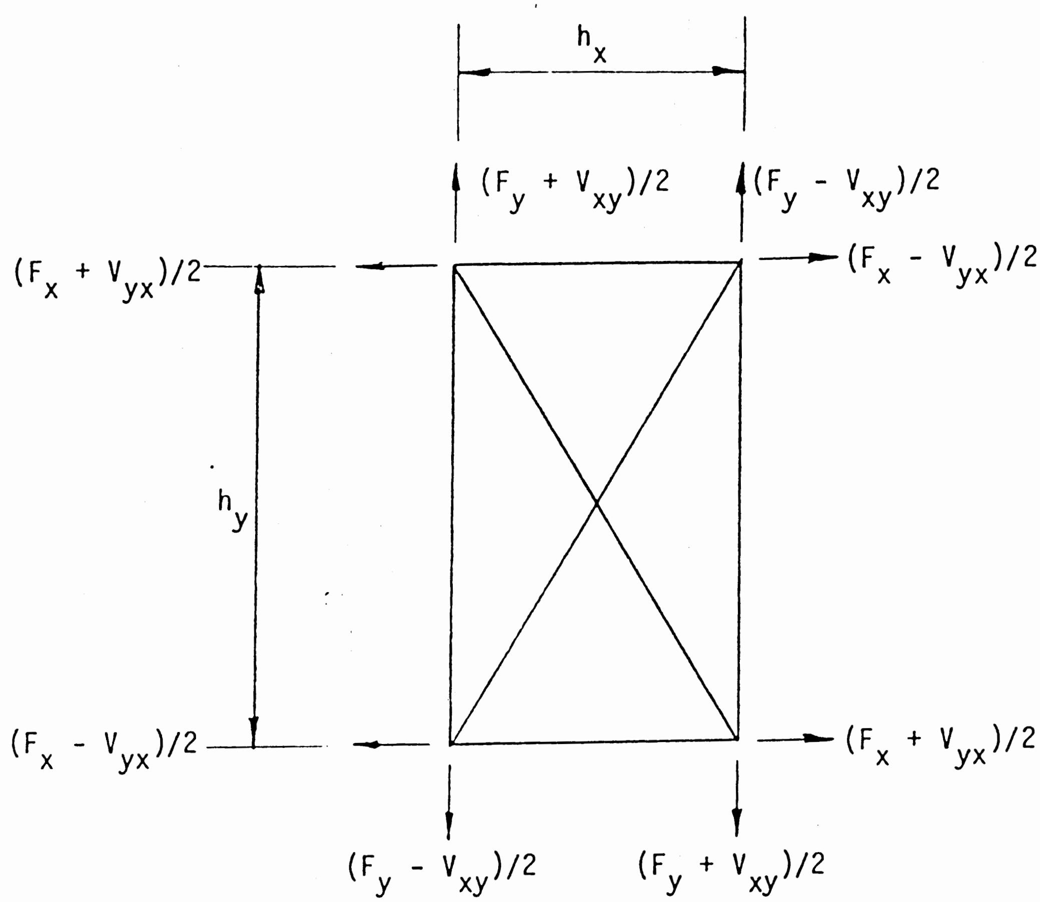


Figure 36. Discrete Element Equivalent Loads (1)

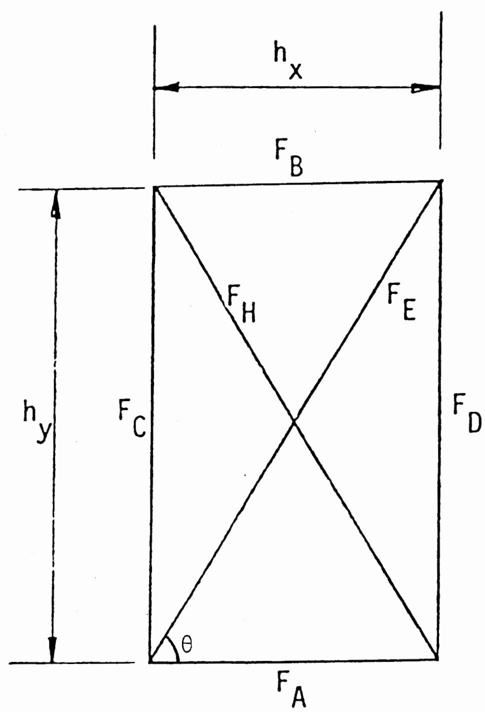


Figure 37. Identification of Membrane Bar Forces (1)

$$[F_A + F_H \cos\theta] = [F_x + V_{yx}]/2 \tag{C.23g}$$

$$[F_D + F_H \sin\theta] = [F_y + V_{xy}]/2 \tag{C.23h}$$

The angle θ is shown in Figure 37. By solving the above equations simultaneously, the plate forces are obtained as follows:

$$F_x = [F_A + F_B] + [F_H + F_E] \cos\theta \tag{C.24a}$$

$$F_y = [F_C + F_D] + [F_H + F_E] \sin\theta \tag{C.24b}$$

$$V_{xy} = [F_H - F_E] \sin\theta \tag{C.24c}$$

$$V_{yx} = [F_H - F_E] \cos\theta \tag{C.24d}$$

Substituting the above quantities into Equation (C.22) yields

$$(\sigma_x)_M = [(F_A + F_B) + (F_H + F_E) \cos\theta]/th_y \tag{C.25a}$$

$$(\sigma_y)_M = [(F_C + F_D) + (F_H + F_E) \sin\theta]/th_x \tag{C.25b}$$

$$(\tau_{xy})_M = [F_H - F_E] \sin\theta/th_y \tag{C.25c}$$

$$(\tau_{yx})_M = [F_H - F_E] \cos\theta/th_x \tag{C.25d}$$

where $(\sigma_x)_M$, $(\sigma_y)_M$, $(\tau_{xy})_M$, and $(\tau_{yx})_M$ are membrane stresses in the plate element.

APPENDIX D

PROPERTIES OF THE COMPOSITE MODEL

The model used to simulate the behavior of the plates in this study is made up of two components, bending and membrane. Depending on the rangel of loading, either bending model alone or both components of the model combined are subjected to increments of the transverse loading. When both components are simultaneously loaded, the model is referred to as the composite model. Figure 38 shows all the lateral and external forces at node i,j of both components of the composite model. To form the stiffness matrix of this model, the equilibrium formulation of node i,j of the discrete-element model in the z direction is developed. This equation is obtained by simply adding the lateral membrane resistance $R_{i,j}$ (Equation C.20) to Equation (B.5) and substituting the incremental load $F_{i,j}$ for $(f_B)_{i,j}$ as follows:

$$\Sigma F_z = 0 = F_{i,j} + V^x_{i,j} + V^y_{i,j} - V^x_{i+1,j} - V^y_{i,j+1}$$

$$- S_{i,j}\, w_{i,j} - CF_{i,j} + R_{i,j} \qquad \text{(D.1)}$$

Equation (D.1) can be written in terms of joint displacements as shown in Appendices B and C. In this form it would be similar to Equation (B.9) except that the coefficients of the equation will include the membrane model bar forces. The equilibrium in the z direction for all joints of this model result in a set of simultaneous equations that can be written symbolically as

$$[(K_M) + (K_B)]\ \{w\} = \{F\} \qquad \text{(D.2)}$$

where

$[(K_M) + (K_B)]$ = lateral stiffness of the composite model;

$\{w\}$ = vector of lateral displacements; and

$\{F\}$ = applied forces.

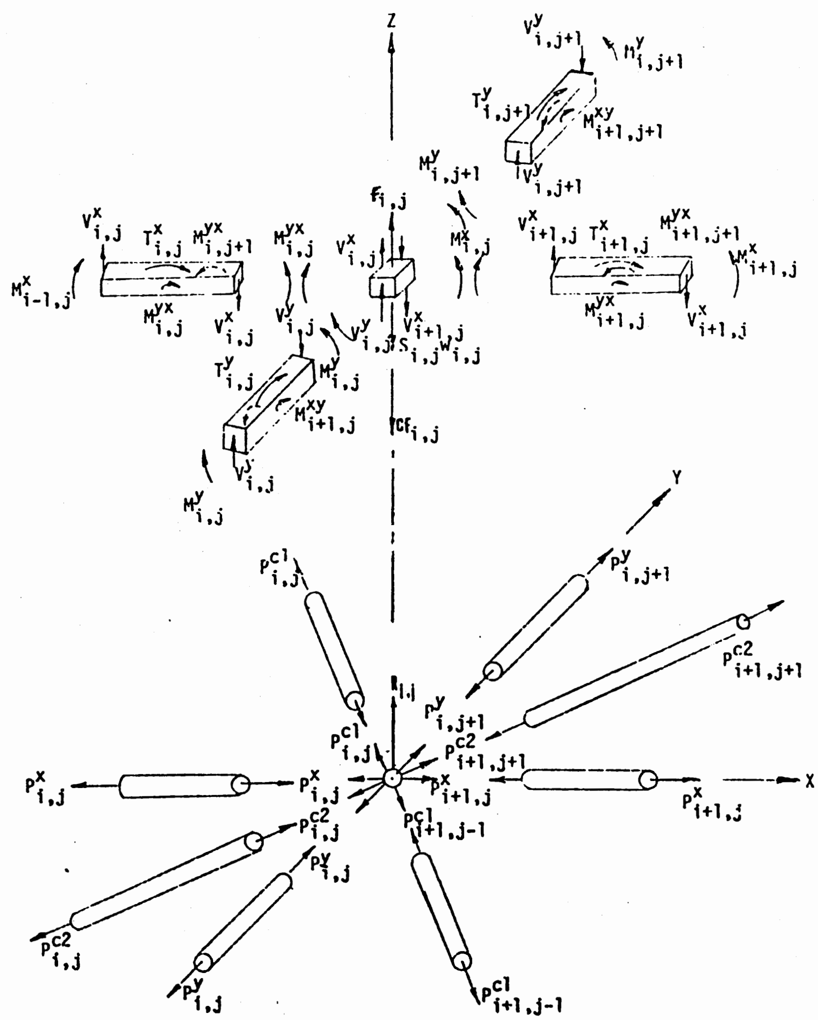


Figure 38. Expanded Joint of the Composite Model

APPENDIX E

GUIDE FOR DATA INPUT

1. Geometry and Boundary Conditions of Plate (2 alphanumeric cards per run)

1	80
1	80

2. Problem Number and Material Description (1 car per run)

Prob. No.	Description of material properties
1 5	11 80

3. Table 1--Control Data (3 cards for each problem)

NCT2	NCT3	NCT4
1 5	10	15

No. of increments in the x and y directions MX	MY	Length of increments HX	HY	Elastic isotropic Poisson's ratio PR	Plate thick-ness THK	Max. No. of Itera-tions NITER	Def. clo-sure for bending model CLOS
1 5	10	20	30	40	50	55	65

Table 1 (Continued)

Yield strain YSTAIN	Yield Stress YILSTZ	Post-elastic modulus ALPHA	No. of loading steps ISYM	Def. closure tolerance for composite model ALDF
1–10	10–20	20–30	30–35	35–45

Table 2--Plate Stiffness (maximum 20 cards)

From sta. IN1	From sta. JN1	Through sta. IN2	Through sta. JN2	Bending stiffness DXN	Bending stiffness DYN	Twisting stiffness CN	Elastic modulus EXN	Elastic modulus EYN
1–5	5–10	10–15	15–20	20–30	30–40	40–50	50–60	60–70

Table 3--Supporting Spring Stiffness (maximum 20 cards)

From sta. IN1	From sta. JN1	Through sta. IN2	Through sta. JN2	Vertical spring SN	In-plane springs SUN	In-plane springs SVN	Rotational springs RXN	Rotational springs RYN
1–5	5–10	10–15	15–20	20–30	30–40	40–50	50–60	60–70

Table 4--Loading (maximum 20 cards)

From sta.		Through sta.		Vertical loading	In-plane loading		External couple	
IN1	JN1	IN2	JN2	QN	PXN	PYN	TXN	TYN
1	5	10	15	20	30	40	50	60

Column boundaries: 1, 5, 10, 15, 20, 30, 40, 50, 60, 70

END OF RUN (one blank card)

Columns 1 to 80

E.1 General Remarks

All the five space data entries are right justified integers.

All the ten space data entries are real variables read by E10.3 format.

E.2 Detail of Input Information

In this section, details of various input items and their units are given. The recommended range for deflection closure tolerances and certain variables are also included.

E.2.1 Description of Geometry, Boundary Conditions, and Material of the Plate

Two cards are used for describing the plate's geometry and boundary conditions.

A single card is needed for problem number and describing the type of the plate's material.

E.2.2 Table 1--Control Data (3 Cards Per Run)

The number of cards in Tables 2 to 4 (NCT2, NCT3, and NCT4) are specified in the first card.

The variables and their units on the second card are

	Length Increment in x-Direction	Length Increment in y-Direction	Thickness
Variables:	H_x	H_y	THK
Units:	in.	in.	in.

MX and MY are the number of increments in the x and y directions. PR is the Poisson ratio of the isotropic plate.

Maximum number of iterations is suggested to be less than 25.

Deformation closure tolerance for the bending model (CLOS) is recommended to be assigned values not more than two-tenths percent of the thickness of the plate.

The variables and their units on the third card are

	Yield Stress	Post-Elastic Modulus	Yield Strain
Variables:	YILSTZ	ALPHA	YSTAIN
Units:	lb/in.^{2}	lb/in.^{2}	in./in.

ISYM is the number of loading steps.

ALDF is deflection closure tolerance for the composite model. It is recommended that ALDF be taken less than one-tenth percent of the thickness of the plate.

E.2.3 Table 2--Plate Stiffness

	Bending Stiffness		Twisting Stiffness	Elastic Modulus	
Variables:	DXN	DYN	CN	EXN	EYN
Units:	lb-in.	lb-in.	lb-in.	lb/in.^{2}	lb/in.^{2}

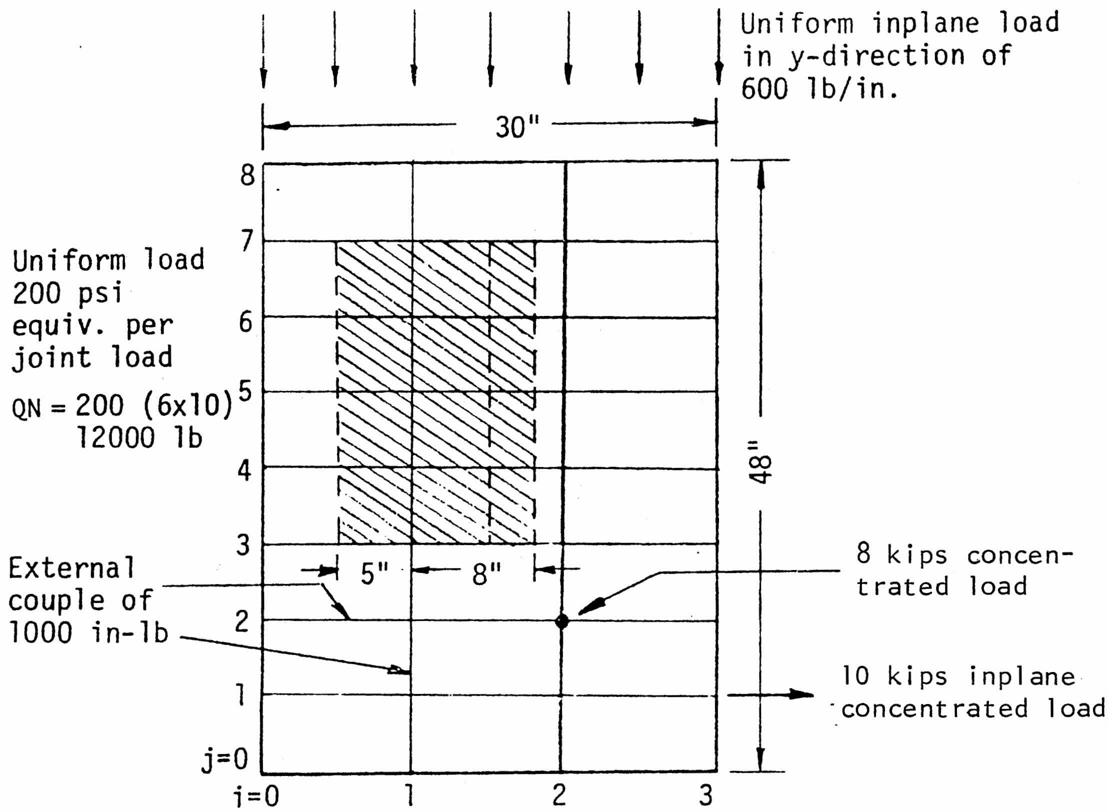
The maximum number of cards in Table 2 is 20.

Data are described by a node coordinate identification as shown in Figure 39.

Bending stiffness is a joint datum.

Twisting stiffness and elastic modulus are area data.

Data may be distributed to every joint in an area by specifying the lower left-hand and upper right-hand coordinates. Quarter-values are automatically placed at corner joints, and half-values are placed at edge joints. For line specification, half-values are placed at the start and end joints. Data for a single point will be identified by placing the same joint coordinates in both the "From sta." and "Through sta." columns.



From Sta. IN1	JN1	Thru Sta. IN2	JN2	DXN	DYN	CN	EXN	EYN
0	0	3	8	1.0E08	1.0E08	8.0E07	3.6E06	3.6E06
				SN	SUN	SVN	RXN	RYN
0	0	3	0	1.0E40	1.0E40	1.0E40	1.0E40	1.0E40
0	0	0	8	1.0E40	1.0E40	1.0E40		
3	0	3	8	1.0E40				
0	8	3	8	1.0E40				
				QN	PXN	PYN	TXN	TYN
2	2	2	2	8.0E03				
1	3	1	7	1.2E04				
2	3	2	7	3.6E03				
3	1	3	1		1.0E04			
0	8	3	8			-6.0E03		
1	2	1	2				1.0E03	1.0E03

Figure 39. Example for Data Input (1)

Coordinates IN2, JN2 must be equal to or greater than coordinates IN1, JN1.

Data on each card are added to preceding card values.

E.2.4 Table 3--Supporting Spring Stiffness

	Vertical Spring	In-Plane Springs		Rotational Springs	
Variables:	SN	SUN	SYN	RXN	RYN
Units:	lb/in.	lb/in.	lb/in.	lb-in./rad	lb-in./rad

The maximum number of cards in Table 3 is 20.

Data are distributed the same way as described in Table 2.

The magnitude of the supporting springs should simulate the real behavior of the supports.

E.2.5 Table 4--Loading System

	Vertical Loading	In-Plane Loading		External Couples	
Variables:	QN	PXN	PYN	TXN	TYN
Units:	lb	lb	lb	lb-in.	lb-in.

The maximum number of cards in Table 4 is 20.

Vertical and in-plane loads are applied directly at joints, and data are distributed the same way as described in Table 2.

External couples are applied to the bar elements left and below station specified.

Sign convention for the loading system is the same as for the plate problem.

APPENDIX F

SAMPLE OUTPUT FOR EXAMPLE PROBLEM 2

In this appendix a sample of the computer output for example problem 2 is presented. The example problem chosen deals with large elastic-plastic deflections of a simply supported thin plate. The plate edges are restrained against in-plane translation. The plate is subjected to uniform lateral pressure.

In the solution procedure the total lateral pressure is applied to the plate in 20 steps. The computer output consists of the results of the analysis of the plate when subjected to these pressure increments. In this appendix the input data and results of the analysis of the plate when subjected to pressure increment q = 14.66 psi are presented in Tables 1 and 2, respectively.

TABLE 1

INPUT DATA

THIN SQUARE PLATE
SIMPLY SUPPORTED

PROB .
616 NO HARDENING

TABLE 1. CONTROL DATA

CARDS TABLE 2	1
CARDS TABLE 3	4
CARDS TABLE 4	1
NUM INCREMENTS MX	8
NUM INCREMENTS MY	8
INCR LENGTH HX	0.250D+01
INCR LENGTH HY	0.250D+01
POISSON RATIO	0.300D+00
YIELD STRAIN	0.360D-02
YIELD STRESSS	0.360D+05
POST YIELD MOD.	0.0
SLAB THICKNESS	0.250D+00
DEF. TOL(BEN)	0.500D-03
DEF. TOL(COM)	0.200D-03
NO. OF LOAD STEPS	20
MAX NUM ITERATION	21

TABLE 2. STIFFNESS DATA FOR PLATE PROBLEM

FROM		THRU		DX	DY	C	EX	EY
0	0	8	8	1.4309D+04	1.4309D+04	1.0016D+04	1.0000D+07	1.0000D+07

TABLE 3. STIFFNESS FOR SUPPORTING SPRINGS

FROM		THRU		S	SU	SV	RX	RY
0	0	8	0	1.0000D+50	1.0000D+50	1.0000D+50	0.0	0.0
8	0	8	8	1.0000D+50	1.0000D+50	1.0000D+50	0.0	0.0
0	0	0	8	1.0000D+50	1.0000D+50	1.0000D+50	0.0	0.0
0	8	8	8	1.0000D+50	1.0000D+50	1.0000D+50	0.0	0.0

TABLE 4. LOAD DATA

FROM		THRU		Q	PX	PY	TX	TY
0	0	8	8	9.1550D+02	0.0	0.0	0.0	0.0

TABLE 2

RESULTS OF ANALYSIS DUE TO APPLICATION OF UNIFORM LATERAL PRESSURE, $q = 14.66$ PSI

THIN SQUARE PLATE
SIMPLY SUPPORTED

PROB (CONTD).
616 NO HARDENING

TABLE 5 RESULTS FOR 20.0PER CENT OF LOAD

I	J	WDEFL	UDEFL	VDEFL	TOTREACT
0	0	0.0	0.0	0.0	2.797D+02
1	0	0.0	0.0	0.0	-1.997D+02
2	0	0.0	0.0	0.0	-3.472D+02
3	0	0.0	0.0	0.0	-4.615D+02
4	0	0.0	0.0	0.0	-5.049D+02
5	0	0.0	0.0	0.0	-4.615D+02
6	0	0.0	0.0	0.0	-3.472D+02
7	0	0.0	0.0	0.0	-1.997D+02
8	0	0.0	0.0	0.0	2.797D+02
0	1	0.0	0.0	0.0	-1.997D+02
1	1	8.804D-02	-4.691D-04	-4.691D-04	1.831D+02
2	1	1.507D-01	-5.200D-04	-2.076D-03	1.830D+02
3	1	1.860D-01	-3.024D-04	-3.369D-03	1.830D+02
4	1	1.973D-01	0.0	-3.833D-03	1.829D+02
5	1	1.860D-01	3.024D-04	-3.369D-03	1.830D+02
6	1	1.507D-01	5.200D-04	-2.076D-03	1.830D+02
7	1	8.804D-02	4.691D-04	-4.691D-04	1.831D+02
8	1	0.0	0.0	0.0	-1.997D+02
0	2	0.0	0.0	0.0	-3.472D+02
1	2	1.507D-01	-2.076D-03	-5.200D-04	1.830D+02
2	2	2.614D-01	-2.364D-03	-2.364D-03	1.831D+02
3	2	3.253D-01	-1.426D-03	-3.950D-03	1.831D+02
4	2	3.459D-01	0.0	-4.535D-03	1.830D+02
5	2	3.253D-01	1.426D-03	-3.950D-03	1.831D+02
6	2	2.614D-01	2.364D-03	-2.364D-03	1.831D+02
7	2	1.507D-01	2.076D-03	-5.200D-04	1.830D+02
8	2	0.0	0.0	0.0	-3.472D+02
0	3	0.0	0.0	0.0	-4.615D+02
1	3	1.860D-01	-3.369D-03	-3.024D-04	1.830D+02
2	3	3.253D-01	-3.950D-03	-1.426D-03	1.831D+02
3	3	4.073D-01	-2.430D-03	-2.430D-03	1.831D+02
4	3	4.339D-01	0.0	-2.810D-03	1.831D+02
5	3	4.073D-01	2.430D-03	-2.430D-03	1.831D+02
6	3	3.253D-01	3.950D-03	-1.426D-03	1.831D+02
7	3	1.860D-01	3.369D-03	-3.024D-04	1.830D+02
8	3	0.0	0.0	0.0	-4.615D+02
0	4	0.0	0.0	0.0	-5.049D+02
1	4	1.973D-01	-3.833D-03	0.0	1.829D+02
2	4	3.459D-01	-4.535D-03	0.0	1.830D+02
3	4	4.339D-01	-2.810D-03	0.0	1.831D+02
4	4	4.627D-01	0.0	0.0	1.831D+02
5	4	4.339D-01	2.810D-03	0.0	1.831D+02
6	4	3.459D-01	4.535D-03	0.0	1.830D+02
7	4	1.973D-01	3.833D-03	0.0	1.829D+02
8	4	0.0	0.0	0.0	-5.049D+02
0	5	0.0	0.0	0.0	-4.615D+02
1	5	1.860D-01	-3.369D-03	3.024D-04	1.830D+02
2	5	3.253D-01	-3.950D-03	1.426D-03	1.831D+02
3	5	4.073D-01	-2.430D-03	2.430D-03	1.831D+02
4	5	4.339D-01	0.0	2.810D-03	1.831D+02
5	5	4.073D-01	2.430D-03	2.430D-03	1.831D+02
6	5	3.253D-01	3.950D-03	1.426D-03	1.831D+02
7	5	1.860D-01	3.369D-03	3.024D-04	1.830D+02
8	5	0.0	0.0	0.0	-4.615D+02
0	6	0.0	0.0	0.0	-3.472D+02
1	6	1.507D-01	-2.076D-03	5.200D-04	1.830D+02
2	6	2.614D-01	-2.364D-03	2.364D-03	1.831D+02
3	6	3.253D-01	-1.426D-03	3.950D-03	1.831D+02
4	6	3.459D-01	0.0	4.535D-03	1.830D+02
5	6	3.253D-01	1.426D-03	3.950D-03	1.831D+02
6	6	2.614D-01	2.364D-03	2.364D-03	1.831D+02
7	6	1.507D-01	2.076D-03	5.200D-04	1.830D+02
8	6	0.0	0.0	0.0	-3.472D+02
0	7	0.0	0.0	0.0	-1.997D+02
1	7	8.804D-02	-4.691D-04	4.691D-04	1.831D+02
2	7	1.507D-01	-5.200D-04	2.076D-03	1.830D+02
3	7	1.860D-01	-3.024D-04	3.369D-03	1.830D+02
4	7	1.973D-01	0.0	3.833D-03	1.829D+02
5	7	1.860D-01	3.024D-04	3.369D-03	1.830D+02
6	7	1.507D-01	5.200D-04	2.076D-03	1.830D+02
7	7	8.804D-02	4.691D-04	4.691D-04	1.831D+02
8	7	0.0	0.0	0.0	-1.997D+02
0	8	0.0	0.0	0.0	2.797D+02
1	8	0.0	0.0	0.0	-1.997D+02
2	8	0.0	0.0	0.0	-3.472D+02
3	8	0.0	0.0	0.0	-4.615D+02
4	8	0.0	0.0	0.0	-5.049D+02
5	8	0.0	0.0	0.0	-4.615D+02
6	8	0.0	0.0	0.0	-3.472D+02
7	8	0.0	0.0	0.0	-1.997D+02
8	8	0.0	0.0	0.0	2.797D+02

NO. OF ITERATIONS 6

PROB (CONTD).
616 NO HARDENING

TABLE 6 BENDING AND TWISTING MOMENTS

I	J	BMX	BMY	TMX	TMY
0	0	0.0	0.0	-3.528D+01	3.528D+01
1	0	-5.647D-03	-1.882D-02	-6.039D+01	6.039D+01
2	0	-1.069D-02	-3.562D-02	-3.927D+01	3.927D+01
3	0	-1.447D-02	-4.823D-02	-1.864D+01	1.864D+01
4	0	-1.592D-02	-5.307D-02	-1.056D-13	1.056D-13
5	0	-1.447D-02	-4.823D-02	1.864D+01	-1.864D+01
6	0	-1.069D-02	-3.562D-02	3.927D+01	-3.927D+01
7	0	-5.647D-03	-1.882D-02	6.039D+01	-6.039D+01
8	0	0.0	0.0	3.528D+01	-3.528D+01
0	1	-1.882D-02	-5.647D-03	-6.039D+01	6.039D+01
1	1	-7.547D+01	-7.547D+01	-1.047D+02	1.047D+02
2	1	-9.018D+01	-1.106D+02	-6.995D+01	6.995D+01
3	1	-8.734D+01	-1.237D+02	-3.385D+01	3.385D+01
4	1	-8.471D+01	-1.267D+02	-1.724D-13	1.724D-13
5	1	-8.734D+01	-1.237D+02	3.385D+01	-3.385D+01
6	1	-9.018D+01	-1.106D+02	6.995D+01	-6.995D+01
7	1	-7.547D+01	-7.547D+01	1.047D+02	-1.047D+02
8	1	-1.882D-02	-5.647D-03	6.039D+01	-6.039D+01
0	2	-3.562D-02	-1.069D-02	-3.927D+01	3.927D+01
1	2	-1.106D+02	-9.018D+01	-6.995D+01	6.995D+01
2	2	-1.390D+02	-1.390D+02	-4.937D+01	4.937D+01
3	2	-1.387D+02	-1.610D+02	-2.488D+01	2.488D+01
4	2	-1.357D+02	-1.669D+02	-1.557D-13	1.557D-13
5	2	-1.387D+02	-1.610D+02	2.488D+01	-2.488D+01
6	2	-1.390D+02	-1.390D+02	4.937D+01	-4.937D+01
7	2	-1.106D+02	-9.018D+01	6.995D+01	-6.995D+01
8	2	-3.562D-02	-1.069D-02	3.927D+01	-3.927D+01
0	3	-4.823D-02	-1.447D-02	-1.864D+01	1.864D+01
1	3	-1.237D+02	-8.734D+01	-3.385D+01	3.385D+01
2	3	-1.610D+02	-1.387D+02	-2.488D+01	2.488D+01
3	3	-1.647D+02	-1.647D+02	-1.294D+01	1.294D+01
4	3	-1.627D+02	-1.724D+02	-1.112D-13	1.112D-13
5	3	-1.647D+02	-1.647D+02	1.294D+01	-1.294D+01
6	3	-1.610D+02	-1.387D+02	2.488D+01	-2.488D+01
7	3	-1.237D+02	-8.734D+01	3.385D+01	-3.385D+01
8	3	-4.823D-02	-1.447D-02	1.864D+01	-1.864D+01
0	4	-5.307D-02	-1.592D-02	-2.780D-14	2.780D-14
1	4	-1.267D+02	-8.471D+01	-1.112D-14	1.112D-14
2	4	-1.669D+02	-1.357D+02	-2.780D-14	2.780D-14
3	4	-1.724D+02	-1.627D+02	-8.340D-14	8.340D-14
4	4	-1.710D+02	-1.710D+02	-3.892D-14	3.892D-14
5	4	-1.724D+02	-1.627D+02	1.668D-14	-1.668D-14
6	4	-1.669D+02	-1.357D+02	5.004D-14	-5.004D-14
7	4	-1.267D+02	-8.471D+01	7.784D-14	-7.784D-14
8	4	-5.307D-02	-1.592D-02	4.170D-14	-4.170D-14
0	5	-4.823D-02	-1.447D-02	1.864D+01	-1.864D+01
1	5	-1.237D+02	-8.734D+01	3.385D+01	-3.385D+01
2	5	-1.610D+02	-1.387D+02	2.488D+01	-2.488D+01
3	5	-1.647D+02	-1.647D+02	1.294D+01	-1.294D+01
4	5	-1.627D+02	-1.724D+02	4.448D-14	-4.448D-14
5	5	-1.647D+02	-1.647D+02	-1.294D+01	1.294D+01
6	5	-1.610D+02	-1.387D+02	-2.488D+01	2.488D+01
7	5	-1.237D+02	-8.734D+01	-3.385D+01	3.385D+01
8	5	-4.823D-02	-1.447D-02	-1.864D+01	1.864D+01
0	6	-3.562D-02	-1.069D-02	3.927D+01	-3.927D+01
1	6	-1.106D+02	-9.018D+01	6.995D+01	-6.995D+01
2	6	-1.390D+02	-1.390D+02	4.937D+01	-4.937D+01
3	6	-1.387D+02	-1.610D+02	2.488D+01	-2.488D+01
4	6	-1.357D+02	-1.669D+02	1.612D-13	-1.612D-13
5	6	-1.387D+02	-1.610D+02	-2.488D+01	2.488D+01
6	6	-1.390D+02	-1.390D+02	-4.937D+01	4.937D+01
7	6	-1.106D+02	-9.018D+01	-6.995D+01	6.995D+01
8	6	-3.562D-02	-1.069D-02	-3.927D+01	3.927D+01
0	7	-1.882D-02	-5.647D-03	6.039D+01	-6.039D+01
1	7	-7.547D+01	-7.547D+01	1.047D+02	-1.047D+02
2	7	-9.018D+01	-1.106D+02	6.995D+01	-6.995D+01
3	7	-8.734D+01	-1.237D+02	3.385D+01	-3.385D+01
4	7	-8.471D+01	-1.267D+02	2.391D-13	-2.391D-13
5	7	-8.734D+01	-1.237D+02	-3.385D+01	3.385D+01
6	7	-9.018D+01	-1.106D+02	-6.995D+01	6.995D+01
7	7	-7.547D+01	-7.547D+01	-1.047D+02	1.047D+02
8	7	-1.882D-02	-5.647D-03	-6.039D+01	6.039D+01
0	8	0.0	0.0	3.528D+01	-3.528D+01
1	8	-5.647D-03	-1.882D-02	6.039D+01	-6.039D+01
2	8	-1.069D-02	-3.562D-02	3.927D+01	-3.927D+01
3	8	-1.447D-02	-4.823D-02	1.864D+01	-1.864D+01
4	8	-1.592D-02	-5.307D-02	1.418D-13	-1.418D-13
5	8	-1.447D-02	-4.823D-02	-1.864D+01	1.864D+01
6	8	-1.069D-02	-3.562D-02	-3.927D+01	3.927D+01
7	8	-5.647D-03	-1.882D-02	-6.039D+01	6.039D+01
8	8	0.0	0.0	-3.528D+01	3.528D+01

PROB (CONTD). .
616 NO HARDENING

TABLE 7 MEMBRANE STRESSES

I.J		MSX	MSY	SHS
1	1	2.066D+03	2.066D+03	-4.035D+02
2	1	3.438D+03	7.764D+03	-2.619D+02
3	1	4.833D+03	1.337D+04	-1.731D+02
4	1	5.650D+03	1.667D+04	-6.084D+01
5	1	5.650D+03	1.667D+04	6.084D+01
6	1	4.833D+03	1.336D+04	1.731D+02
7	1	3.438D+03	7.760D+03	2.619D+02
8	1	2.066D+03	2.066D+03	4.035D+02
1	2	7.763D+03	3.439D+03	-2.624D+02
2	2	7.966D+03	7.968D+03	5.902D+02
3	2	8.374D+03	1.334D+04	5.861D+02
4	2	8.650D+03	1.660D+04	2.282D+02
5	2	8.650D+03	1.659D+04	-2.282D+02
6	2	8.374D+03	1.334D+04	-5.861D+02
7	2	7.966D+03	7.966D+03	-5.902D+02
8	2	7.763D+03	3.438D+03	2.624D+02
1	3	1.337D+04	4.836D+03	-1.736D+02
2	3	1.334D+04	8.377D+03	5.864D+02
3	3	1.315D+04	1.315D+04	6.108D+02
4	3	1.301D+04	1.617D+04	2.417D+02
5	3	1.301D+04	1.617D+04	-2.417D+02
6	3	1.315D+04	1.315D+04	-6.108D+02
7	3	1.334D+04	8.376D+03	-5.864D+02
8	3	1.337D+04	4.835D+03	1.736D+02
1	4	1.667D+04	5.654D+03	-6.110D+01
2	4	1.659D+04	8.653D+03	2.284D+02
3	4	1.617D+04	1.302D+04	2.418D+02
4	4	1.585D+04	1.585D+04	9.668D+01
5	4	1.585D+04	1.585D+04	-9.668D+01
6	4	1.617D+04	1.302D+04	-2.418D+02
7	4	1.659D+04	8.654D+03	-2.284D+02
8	4	1.667D+04	5.654D+03	6.110D+01
1	5	1.667D+04	5.654D+03	6.110D+01
2	5	1.659D+04	8.652D+03	-2.284D+02
3	5	1.617D+04	1.302D+04	-2.418D+02
4	5	1.585D+04	1.585D+04	-9.668D+01
5	5	1.585D+04	1.585D+04	9.668D+01
6	5	1.617D+04	1.302D+04	2.418D+02
7	5	1.659D+04	8.654D+03	2.284D+02
8	5	1.667D+04	5.654D+03	-6.110D+01
1	6	1.337D+04	4.835D+03	1.736D+02
2	6	1.334D+04	8.375D+03	-5.864D+02
3	6	1.315D+04	1.315D+04	-6.108D+02
4	6	1.301D+04	1.617D+04	-2.417D+02
5	6	1.301D+04	1.617D+04	2.417D+02
6	6	1.315D+04	1.315D+04	6.108D+02
7	6	1.334D+04	8.378D+03	5.864D+02
8	6	1.337D+04	4.836D+03	-1.736D+02
1	7	7.763D+03	3.438D+03	2.624D+02
2	7	7.966D+03	7.966D+03	-5.902D+02
3	7	8.374D+03	1.334D+04	-5.861D+02
4	7	8.650D+03	1.659D+04	-2.282D+02
5	7	8.650D+03	1.660D+04	2.282D+02
6	7	8.374D+03	1.334D+04	5.861D+02
7	7	7.966D+03	7.967D+03	5.902D+02
8	7	7.763D+03	3.439D+03	-2.624D+02
1	8	2.066D+03	2.066D+03	4.035D+02
2	8	3.438D+03	7.763D+03	2.619D+02
3	8	4.833D+03	1.337D+04	1.731D+02
4	8	5.650D+03	1.667D+04	6.084D+01
5	8	5.650D+03	1.667D+04	-6.084D+01
6	8	4.833D+03	1.336D+04	-1.731D+02
7	8	3.438D+03	7.761D+03	-2.619D+02
8	8	2.066D+03	2.066D+03	-4.035D+02

APPENDIX G

A LISTING OF THE COMPUTER PROGRAM

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      IMPLICIT REAL*8(A-H,O-Z)
C--------PAGE 1
C-----DIMENSION STATEMENTS
      DIMENSION    A(23,25),        B(23,23,25),        BMX(27,27),
     1             BMY(27,27),      C(23,23,25),        CX(27,27),
     2                              DX(27,27),          DY(27,27),
     3             S(27,27),        TX(27,27),          TY(27,27),
     4             Q(27,27),        W1(27,1),           W2(23,1)
      DIMENSION    RXN(20), RYN(20), RX(27,27), RY(27,27)
      DIMENSION    AN1(40),         AN2(35),            DP(6)
      DIMENSION    IN1(20), IN2(20), JN1(20), JN2(20), DXN(20), DYN(20),
     1             QN(20), SN(20), CXN(20), TXN(20), TYN(20),
     2             PXN(20), PYN(20), SUN(20), SVN(20)
      DIMENSION    PXX(27,27), PYY(27,27), PZZ(27,27), EXN(20), EYN(20)
      DIMENSION    QQ(27,27), QQ1(27,27), PX(27,27), PX1(27,27),
     1             PY(27,27),PY1(27,27),DSX(27,27),DSY(27,27),SHS(27,27)
      DIMENSION    PC1(27,27), PC2(27,27), UU(27,27), VV(27,27)
      DIMENSION PD1(27,27), PD2(27,27)
      DIMENSION PHIX1(27,27),PHIY1(27,27), PHIXY1(27,27),EPSMX1(27,27),
     1 EPSMY1(27,27),EPMXY1(27,27),EXS1(27,27),
     1 EYS1(27,27),NNXB(27,27),NNYB(27,27),NNXS(27,27),NNYS(27,27)
      DIMENSION SIGX(21),SIGY(21),TAU(21),SF(21),SFF(21),F(21),
     1 ASF(27,27,21),ASFF(27,27,21),AF(27,27,21)
      DIMENSION  EPSX(21),EPSY(21),EPSXY(21),EPPX(21),EPPY(21),EPPXY(21)
      DIMENSION EEX(27,27),EEY(27,27),ABX(27,27),ABY(27,27),ABG(27,27)
      DIMENSION    BG(27,27)
      DIMENSION    QQ11(27,27)
      DIMENSION    PXY1(27,27)
      DIMENSION    PXY2(27,27)
      DIMENSION    PMOMX(27,27)
      DIMENSION    Q3(27,27),Q4(27,27)
      DIMENSION    QQ2(27,27)
      DIMENSION    PZ1(27,27),PZ2(27,27)
      DIMENSION    GB(27,27),GS(27,27)
      DIMENSION    XC(27,27)
      DIMENSION    XWW(27,27) ,XUU(27,27),XVV(27,27)
            REAL*8 NEUO,NUXB1,NUYB1,NUXS1,NUYS1
            REAL*8 NNXB,NNYB,NNXS,NNYS
      DATA ITEST/4H    /
C
C-----COMMON CARDS
C
      COMMON PR
      COMMON/INCR/MX,MY,MXP2,MXP3,MXP4,MXP5,MXP7,
     1            MYP2,MYP3,MYP4,MYP5,MYP7
      COMMON/MATR/ AA1(23),AA2(23,3),AA3(23,5),AA4(23,3),
     1 AA5(23),AA6(23),AA(23,1),A1(23,1),A2(23,1),
     2 BB(23,23),BB1(23,23),BB2(23,23),CC(23,23),
     3 CC1(23,23),CC2(23,23),AAUG(23,23,2),D(23,23),E(23,23),D1(23,23)
      COMMON/DAT/CORDX(441),CORDY(441),CORDZ(441),
     1 EC(27,27),EX(27,27),EY(27,27),AE(1640),JT(1640),KT(1640),
     2  AX(27,27),AY(27,27),AC(27,27)
      COMMON/PLANE/PX2(27,27),PY2(27,27),W(27,27),SU(27,27),SV(27,27),
     1 U(27 27),V(27,27)
            COMMON ALPHA
      COMMON/ZAP/ZEPX(17 17,21),ZEPY(17 17,21),ZEPXY(17 17,21)
     1RAMP(17 17 21)


C
C-----FORMAT STATEMENTS
C
    1 FORMAT(3E10.3,I5,E10.3)
    2 FORMAT(/,' Y-STRAIN   Y-STRESS   2ND SLOPE  LOAD STEP  TOL')
   11 FORMAT(1H1,'I-----TRIM')
   12 FORMAT( 20A4)
   13 FORMAT( 5X,20A4)
   14 FORMAT(A4,A1,5X,35A2)
   15 FORMAT(///10H    PROB , /5X, A4,A1, 5X, 35A2)
   16 FORMAT(///17H    PROB (CONTD), /5X,A4,A1,5X,35A2)
   19 FORMAT(5X, 1H   )
   20 FORMAT(5I5,4E10 3, 2A4)
   21 FORMAT(2I5,4E10 3,I5,E10 3)
   23 FORMAT(   4(2X,I3),6E10 4)
   24 FORMAT( 4(2X I3),6E10 4)
   30 FORMAT(//30H TABLE 1. CONTROL DATA                          . /
     1/23H         CARDS TABLE 2    ,43X,I3,   /
     1 23H         CARDS TABLE 3    ,43X,I3,   /
     3 23H         CARDS TABLE 4    ,43X,I3,   /
     4 26H         NUM INCREMENTS MX  , 40X,I3 ,/
     5 28H         NUM INCREMENTS MY  , 38X,I3 ,/
     6 28H         INCR LENGTH  HX     ,31X,  E10.3,/
     6 27H         INCR LENGTH  HY     ,32X,  E10.3,/
     9 26H         POISSON RATIO      ,33X,E10.3, /
     9 26H         YIELD STRAIN       ,33X,E10.3, /
     9 26H         YIELD STRESSS      ,33X,E10.3, /
     9 26H         POST YIELD MOD.    ,33X,E10.3, /
     9 24H         SLAB THICKNESS     ,35X,E10.3, /
     1 27H         DEF. TOL(BEN)      ,32X,E10.3,/
     1 27H         DEF. TOL(COM)      ,32X,E10.3,/
     3 28H         NO. OF LOAD STEPS  ,38X,I3,/
     3 28H         MAX NUM ITERATION  ,38X,I3,/
     2 30H                              ,35X,E10.3)
   33 FORMAT(//50H TABLE 2. STIFFNESS DATA FOR PLATE PROBLEM          ,/
     2       /50H        FROM     THRU        DX          DY          C
     3        45H      EX        EY                                   ,/)
   37 FORMAT(//50H TABLE 3. STIFFNESS FOR SUPPORTING SPRINGS          ,/
     2       /50H        FROM     THRU        S           SU          SV
     3        45H        RX       RY                                  ,/)
   38 FORMAT(//25H TABLE 4. LOAD DATA        ,/
     1/70H      FROM     THRU      Q        PX         PY          TX
     2     TY    ,/)
   39 FORMAT(//26H      TABLE 5. RESULTS FOR ,F5.1,'PER CENT OF LOAD',///
     1        50H      I,J     WDEFL           UDEFL       VDEFL
     2        20H TOTREACT          /)
   40 FORMAT(//45H      TABLE 6. BENDING AND TWISTING MOMENTS       ,///
     1        50H      I,J     BMX             BMY       TMX
     2        20H   TMY             ,/)
   41 FORMAT(//48H     TABLE 7.           MEMBRANE STRESSES         ,///
     1        48H      I,J      MSX            MSY          SHS     ,/)
   43 FORMAT(5X,  2(1X,I3,I3),1P6E11.4)
   44 FORMAT(5X, 2( 1X,I3,I3),1P6E11.4)
   45 FORMAT(7X,I2,I3,1P9E12.3)
   46 FORMAT(///7X,' NO. OF ITERATIONS ', I5)
   49 FORMAT(//7X,' PER CENT ERROR IN CLOSURE ', E10.3)
 1996 FORMAT(2I5,3F11.7,F11.2)
22112 FORMAT(10F10.3)
      L1=23
      READ 12, (AN1(N), N=1,40)
 1010 READ 14, NPROB, (AN2(N), N=1,35)
         IF(NPROB-ITEST)1020,9990,1020
 1020 PRINT 11
      PRINT 13,(AN1(N),N=1,40)
      PRINT 15,NPROB, (AN2(N),N=1,35)
C
C-----INPUT TABLE 1
C
      READ 20, NCT2,NCT3,NCT4
      READ 21,MX,MY,HX,HY,PR,THK,NITERA,CLOS
      READ 1, YSTAIN, YILSTZ, ALPHA , ISYM,ALDF
      SYM=ISYM
      KSY=SYM
      ITYPE=0
      PRINT 30,NCT2,NCT3,NCT4,MX,MY,HX,HY,PR,YSTAIN,YILSTZ,ALPHA,THK,
     1CLOS,ALDF,KSY,NITERA
            NEUO=PR
            XM=YILSTZ/YSTAIN
            ISYM=SYM
            ISYM1=ISYM+1
            TH=THK
            KKX=1
            XKAM=1.
            KASH=1
            XAY=0.
            CHECK=0.
            FIN=0.
            HA=.7
            HN=.3
            XMART=1.
            MIZ=1
```

```
      KASN=1
      JIMM=1
C
C     --COMPUTE FOR CONVENIENCE
C
        MX2P4=MX/2+4
        MY2P4=MY/2+4
        MX2=MX/2
        MY2=MY/2
        MXP1=MX+1
        MXP2=MX+2
        MXP3=MX+3
        MXP4=MX+4
        MXP5=MX+5
        MXP7=MX+7
        MYP1=MY+1
        MYP2=MY+2
        MYP3=MY+3
        MYP4=MY+4
        MYP7=MY+7
        MYP5=MY+5
        HYDHX3=HY/HX**3
        HXDHY3=HX/HY**3
        PDHXHY=PR/(HY*HX)
        OOHXHY=1 O/(HY*HX)
        ODHX=1.O/HX
        ODHY=1.O/HY
        ODHX2=1.O/HX**2
        ODHY2=1.O/HY**2
        WMAX1=0.0
        WMAX2=0.0
        WDIFF=0.0
        ITERA=0
        ITREW=0
        BETA=0 10
        ITE=0
        NJT=MXP1*MYP1
        NMEM=4*MX*MY+MX+MY
        HZ=DSQRT(HX*HX+HY*HY)
   DO 105 J=1,MYP7
   DO 100 I=1,MXP7
        BMX(I,J)=0.0
        BMY(I,J)=0.0
        DX(I,J)=0 0
        DY(I,J)=0.0
        S(I,J)=0 0
        Q(I,J)=0.0
        CX(I,J)=0 0
        TX(I,J)=0 0
        TY(I,J)=0 0
        PX(I,J)=0.0
        PY(I,J)=0 0
        SU(I,J)=0.0
        SV(I,J)=0.0
        U(I,J)=0.0
        V(I,J)=0 0
        W(I,J)=0.0
        EX(I,J)=0.0
        EY(I,J)=0 0
        EC(I,J)=0.0
        PXX(I,J)=0.0
        PYY(I,J)=0 0
        PZZ(I,J)=0 0
        PX2(I,J)=0.0
        PX1(I,J)=0 0
        PY1(I,J)=0.0
        PY2(I,J)=0 0
        QQ(I,J)=0.0
        QQ1(I,J)=0 0
        RX(I,J)=0.0
        RY(I,J)=0.0
        SHS(I,J)=0 0
        DSX(I,J)=0 0
        DSY(I,J)=0 0
        AX (I,J)=0.0
        AC (I,J)=0.0
        AY (I,J)=0.0
        PD1(I,J)=0.0
        PD2(I,J)=0.0
        PC1(I,J)=0.0
        PC2(I,J)=0.0
        UU (I,J)=0.0
        VV (I,J)=0.0
        PXY1(I,J)=0.
        PXY2(I,J)=0.
        EXS1(I,J)=0.
        EYS1(I,J)=0.
        NNXB(I,J)=0.
        NNYB(I,J)=0.
        NNXS(I,J)=0.
        NNYS(I,J)=0.
        GB(I,J)=0.0
        GS(I,J)=0.
        QQ2(I,J)=0.
        EEX(I,J)=0.
        EEY(I,J)=0.
        ABX(I,J)=0.
        ABY(I,J)=0.
        ABG(I,J)=0.
        PZ1(I,J)=0
        PZ2(I,J)=0
        Q3(I,J)=0.
        Q4(I,J)=0.
        XWW(I,J)=0.
        XVV(I,J)=0.
        XUU(I,J)=0
  100 CONTINUE
  105 CONTINUE
           ICO=0
           LAMB=5
           FINDA=0.
           LAND=0
      DO 120 NJ=1,NJT
              CORDY(NJ)=0.0
              CORDX(NJ)=0.0
              CORDZ(NJ)=0.0
  120 CONTINUE
      DO 140 J=1,2
      DO 135 II=1,MXP3
              A(II,J)=0.0
      DO 130 I=1,MXP3
              B(I,II,J)=0.0
              C(I,II,J)=0.0
  130 CONTINUE
  135 CONTINUE
 140      CONTINUE
C
C-----INPUT TABLE 2
C
      IF(1.GT.NCT2) GO TO 220
      PRINT 33
      DO 210 L=1,NCT2
      READ 23,IN1(L),JN1(L),IN2(L),JN2(L),DXN(L),DYN(L),
     1CXN(L),EXN(L),EYN(L)
      PRINT 43,IN1(L),JN1(L),IN2(L),JN2(L),DXN(L),DYN(L),
     1CXN(L),EXN(L),EYN(L)
      IF(ITYPE.NE.  1) GO TO 205
              DXN(L)=EXN(L)
              DYN(L)=EYN(L)
  205 CONTINUE
  210 CONTINUE
      CALL INTERP (IN1,JN1,IN2,JN2,DXN,NCT2,DX,1,0,0, 27,27,0,0)
      CALL INTERP (IN1,JN1,IN2,JN2,DYN,NCT2,DY, 1 ,0,0.  27.  27. 0,0)
      CALL INTERP (IN1,JN1,IN2,JN2,CXN,NCT2,CX, 0,0,1.   27.  27. 0,0)
      CALL INTERP (IN1,JN1,IN2,JN2,EXN,NCT2,EX, 0,0,1.   27.  27. 0,0)
      CALL INTERP (IN1,JN1,IN2,JN2,EYN,NCT2,EY, 0,0,1.   27.  27. 0,0)
  220 CONTINUE
           DO 225 I=1,MXP7
           DO 225 J=1, MYP7
           EC(I,J)=(HX*HX*EX(I,J)+HY*HY*EY(I,J))/(HZ*HZ)
  225 CONTINUE
```

```
C-----INPUT TABLE 3
C
C
      IF( 1.GT NCT3 ) GO TO 240
      PRINT 37
            DO 230 L= 1, NCT3
      READ 24, IN1(L),JN1(L),IN2(L),JN2(L),
     1          SN(L), SUN(L), SVN(L), RXN(L), RYN(L)
      PRINT 44,IN1(L),JN1(L),IN2(L),JN2(L),
     1          SN(L), SUN(L), SVN(L), RXN(L), RYN(L)
  230 CONTINUE
      CALL INTERP (IN1,JN1,IN2,JN2, SN,NCT3,S, 1.0,0, 27.  27.0.0 )
      CALL INTERP (IN1,JN1,IN2,JN2,SUN,NCT3,SU,1.0,0, 27.  27.0.0 )
      CALL INTERP (IN1,JN1,IN2,JN2,SVN,NCT3,SV,1.0,0, 27.  27.0.0 )
      CALL INTERP (IN1,JN1,IN2,JN2,RXN,NCT3,RX, 1.0,0,27,27.0.0 )
      CALL INTERP (IN1,JN1,IN2,JN2,RYN,NCT3,RY, 1.0,0,27,27.0.0 )
  240 CONTINUE
            DO 2001 K=1,21
            DO 2001 I=1,MXP7
            DO 2001 J=1,MYP7
            ASF(I,J,K)=EX(I,J)
            AF(I,J,K)=ASF(I,J,K)*YSTAIN
            ASFF(I,J,K)=NEUO
            ZEPX (I,J,K)=0
            ZEPY (I,J,K)=0
            ZEPXY(I,J,K)=0
            RAMP(I,J,K)=0
 2001 CONTINUE
C
C-----INPUT TABLE 4
C
            IF(1 GT NCT4 ) GO TO 260
      PRINT 38
            DO 250 L=1, NCT4
      READ 24, IN1(L), JN1(L), IN2(L), JN2(L),
     1         QN(L), PXN(L), PYN(L), TXN(L), TYN(L)
      PRINT 44,IN1(L), JN1(L), IN2(L), JN2(L),
     1         QN(L)  PXN(L)  PYN(L), TXN(L), TYN(L)
  250 CONTINUE
      CALL INTERP (IN1, JN1, IN2, JN2, QN ,NCT4, Q , 1.0.0.27.27.0.0)
      CALL INTERP (IN1, JN1, IN2, JN2,PXN ,NCT4,PX , 1.0.0.27.27.0.0)
      CALL INTERP (IN1, JN1, IN2, JN2,PYN ,NCT4,PY , 1.0.0.27.27.0.0)
      CALL INTERP (IN1, JN1, IN2, JN2,TXN ,NCT4,TX , 0.1.0.27.27.1.0)
      CALL INTERP (IN1  JN1  IN2, JN2,TYN ,NCT4,TY , 0.1.0.27.27.0.1)
  260 CONTINUE
            DO 1511 J=1,MYP7
            DO 1511 I=1,MXP7
            EYS1(I,J)=EY(I,J)
            EXS1(I,J)=EX(I,J)
            NNXB(I,J)=PR
            NNYB(I,J)=PR
            NNXS(I,J)=PR
            NNYS(I,J)=PR
            GB(I,J)=EX(I,J)/(2.*(1.+PR))
            GS(I,J)=EX(I,J)/(2 *(1 +PR))
 1511       CONTINUE
            DO 12021 I=5,MXP3
            DO 12021 J=5,MXP3
            IF(S(I,J)  LT  1 OE+19) GO TO 12021
            CMP= .25
            DX(I,J)=CMP*DX(I,J)
            DY(I,J)=CMP*DY(I J)
12021       CONTINUE

C     CALCULATE CROSS SECTION AREA OF MEMBRANE BARS

            DO 270 J=5,MYP4
            DO 270 I= 5,MXP4
                AX(I,J)=THK*(HY*HY-PR*HX*HX*EY(I,J)/EX(I,J))
     1         /(2 *HY*(1 -PR*PR))
                AY(I,J)= THK*(HX*HX-PR* HY*HY*EX(I,J)/EY(I,J))
     1         /(2.*HX*(1.-PR*PR))
                 AC(I,J)= THK*PR*(HZ**3)*EY(I,J)/(EC(I,J)*
     2        2.*HX*HY*(1.-PR*PR))
  270 CONTINUE
            IPL=0
  380 CONTINUE
            DO 281 I=4,MXP4
            DO 281 J=4,MYP4
                XUU(I,J)=PX(I,J)
                XVV(I,J)=PY(I,J)
                PX(I,J)=0.0
                PY(I,J)=0.0
  281 CONTINUE
C
C-----PLACE SPRING AT MESH POINTS BEYOND BOUNDARIES OF REAL SLAB TO MAKE
C     SOLUTION OF NON-RECTANGULAR SLABS OR SLABS WITH HOLES POSSIBLE.
C
C     SET INCREMENTAL LOAD VALUES.
C
            DO 400 J=3,MYP5
            DO 395 I=3,MXP5
              QQ(I,J)=Q(I,J)- ODHX*(TX(I,J)-TX(I+1,J))
     1        -ODHY*(TY(I,J)-TY(I,J+1))
            QQ(I,J)=QQ(I,J)/SYM
            QQ11(I,J)=QQ(I,J)
              QQ1(I,J)=BETA*QQ(I,J)
              SUM=DX(I-1,J)+DX(I,J)+DX(I+1,J)
     1   +    DY(I,J-1)+DY(I,J)+DY(I,J+1)
      IF(SUM)395,390,395
  390         S(I,J)=1.0E+20
  395 CONTINUE
  400 CONTINUE
C
C-----CHECK FOR APPLIED INPLANE LOADS
C
            DO 401 I=1,NCT4
      IF(PXN(I) .NE. 0.0) JIMM=2
      IF(PYN(I) .NE. 0.0) JIMM=2
      IF(JIMM .EQ. 2) GO TO 24116
  401 CONTINUE
            DO 74001 J=4,MYP4
            DO 74001 I=4,MXP4
            PMOMX(I,J)=BMX(I,J)
74001       CONTINUE
C
C-----BENDING SOLUTION ( ALL INPLANE FORCES ARE ZERO)
C
C
C-----FORM SUB -MATRICES.
            DO 600 J=3,MYP5
            DO 500 I=3,MXP5
              II=I-2
              AA1(II)=DY(I,J-1)*HXDHY3
     1        -.25*ODHY2* RY(I,J-1)
            AA2(II,2)=-2.*(ODHXHY*(NNYB(I,J)*DX(I,J)+NNXB(I,J-1)*DY(
     1      I,J-1))
     1        +HXDHY3 *(DY(I,J-1)+DY(I,J)) )
     2        +ODHXHY*(-CX(I,J)-CX(I+1,J)
     3        -CX(I,J)-CX(I+1,J))-ODHY*PY1(I,J)
      IF(II-1) 402,403,402
  402 IF(II-MXP3) 407,408,407
  403       AA3(II,3)=HYDHX3*DX(I+1,J)+0.25*ODHX2*RX(I+1,J)
      GO TO 404
  408       AA3(II,3)=HYDHX3*DX(I-1,J)+.25*ODHX2*RX(I-1,J)
      GO TO 404
  407       AA3(II,3)=HYDHX3*(DX(I-1,J)+4.0*DX(I,J)
     1        +DX(I+1,J))+HXDHY3*(DY(I,J-1)+4.0
     2      *DY(I,J)+DY(I,J+1))+ODHXHY*4.*(NNYB(I,J)*DX(I,J)+NNXB(I,J)*
     3      DY(I,J))+ODHXHY
     4        *(CX(I,J)+CX(I,J+1)+CX(I+1,J)
     5        +CX(I+1,J+1)+CX(I,J)+CX(I+1,J)
     6        +CX(I,J+1)+CX(I+1,J+1))+ ODHX
     7       *(PX1(I,J)+PX1(I+1,J))+ODHY
     8        *(PY1(I,J)+PY1(I,J+1))+S(I,J)
     9        +.25*ODHX2*(RX(I-1,J)+RX(I+1,J))
     A        +.25*ODHY2*(RY(I,J-1)+RY(I,J+1))
     B        +(PC1(I,J)+PC1(I+1,J-1)+PC2(I,J)+PC2(I+1,J+1))/HZ
```

```
  404 IF(AA3(II,3) .EQ. 0.0) AA3(II,3)=1.0
          AA4(II,2) =-2.0*(HXDHY3*(DY(I,J)+DY(I,J+1))
     1      +ODHXHY*(NNYB(I,J)*DX(I,J)+NNXB(I,J+1)*DY(I,J+1)))
     2      +ODHXHY*(-CX(I,J+1)-CX(I+1,J+1)
     3      -CX(I,J+1)-CX(I+1,J+1))-ODHY
     4      *PY1(I,J+1)
          AA5(II)=HXDHY3*DY(I,J+1)
     1        -.25*ODHY2*RY(I,J+1)
          AA6(II)=QQ1(I,J)
      IF(II-1) 410,410,405
  405       AA2(II,1)=ODHXHY*(NNYB(I-1,J)*DX(I-1,J)+NNXB(I,J-1)*DY(I,J-1)
     1      )+
     !                 ODHXHY*(CX(I,J)+CX(I,J))
     2                 -PC2(I,J)/HZ
            AA3(II,2)=-2.0*(HYDHX3*(DX(I-1,J)+DX(I,J))
     1      +ODHXHY*(NNYB(I-1,J)*DX(I-1,J)+NNXB(I,J)*DY(I,J)) )
     2        +ODHXHY*(-CX(I,J)-CX(I,J+1)
     3        -CX(I,J)-CX(I,J+1))-ODHX*PX1(I,J)
            AA4(II,1)=ODHXHY*(NNYB(I-1,J)*DX(I-1,J)+NNXB(I,J+1)*DY(I,J+1)
     1      )
     !        +ODHXHY*(CX(I,J+1)+CX(I,J+1))
     2        -PC1(I,J)/HZ
  410 IF(II-2)430,430,420
  420       AA3(II,1)= DX(I-1,J)*HYDHX3
     1        -.25*ODHX2*RX(I-1,J)
  430 IF(II-MXP3) 440,450,450
  440       AA2(II,3)=ODHXHY*(NNYB(I+1,J)*DX(I+1,J)+NNXB(I,J-1)*DY(I,J-1)
     1      )
     1        +ODHXHY*(CX(I+1,J)+CX(I+1,J))
     2        -PC1(I+1,J-1)/HZ
            AA3(II,4)=-2.0*(HYDHX3*(DX(I,J)+DX(I+1,J))
     1      +ODHXHY*(NNYB(I+1,J)*DX(I+1,J)+NNXB(I,J)*DY(I,J))          )
     2        +ODHXHY*(-CX(I+1,J)-CX(I+1,J+1)
     3        -CX(I+1,J)-CX(I+1,J+1))-ODHX
     4        *PX1(I+1,J)
            AA4(II,3)=ODHXHY*(NNYB(I+1,J)*DX(I+1,J)+NNXB(I,J+1)*DY(I,J+1)
     1      )
     1        +ODHXHY*(CX(I+1,J+1)+CX(I+1,J+1))
     2        -PC2(I+1,J+1)/HZ
  450 IF(II+1-MXP3)460,500,500
  460         AA3(II,5)= HYDHX3*DX(I+1,J)
     1        -.25*ODHX2*RX(I+1,J)
  500 CONTINUE
C
C-----BEGIN MAIN SOLUTION
C
            DO 515 I=1,MXP3
              A1(I,1)=A(I,J-1)
              A2(I,1)=A(I,J-2)
            DO 510 K=1,MXP3
              BB1(I,K)=B(I,K,J-1)
              BB2(I,K)=B(I,K,J-2)
              CC1(I,K)=C(I,K,J-1)
              CC2(I,K)=C(I,K,J-2)
  510 CONTINUE
  515 CONTINUE
            JJ=J
      CALL  MATRIX(L1,JJ,MXP3,MY)
            DO 585 I=1,MXP3
            A(I,J)=AA(I,1)
            DO 580 K=1,MXP3
            B(I,K,J)=BB(I,K)
            C(I,K,J)=CC(I,K)
  580 CONTINUE
  585 CONTINUE
  600 CONTINUE
C
C-----COMPUTE LATERAL DEFLECTION
C
            DO 650 LL=3,MYP5
              J=MY+8-LL
            DO 625 I=1,MXP3
              W1(I,1)=W(I,J+1)
              W2(I,1)=W(I,J+2)
              AA(I,1)=A(I,J)
            DO 620 K=1,MXP3
              BB(I,K)=B(I,K,J)
              CC(I,K)=C(I,K,J)
  620 CONTINUE
  625 CONTINUE
      CALL MATMPY (L1,MXP3,1,BB,W1,A1)
      CALL MATMPY (L1,MXP3, 1,CC,W2,A2)
            DO 630 I=1,MXP3
              W(I,J)=AA(I,1)+A1(I,1)+A2(I,1)
  630 CONTINUE
  650 CONTINUE
            W(1,3)=2.*W(1,4)-W(1,5)
            W(MXP3,3)=2.*W(MXP3,4)-W(MXP3,5)
            W(1,MYP5)=2.*W(1,MY+4)-W(1,MY+3)
            W(MXP3,MYP5)=2.*W(MXP3,MY+4)-W(MXP3,MY+3)
            DO 665 J=3,MYP5
            DO 660 I=3,MXP5
              II=MXP5+3-I
              W(II,J)=W(II-2,J)
  660 CONTINUE
  665 CONTINUE
            DO 670 J=3,MYP5
              W(1,J)=0.0
              W(2,J)=0.0
              W(MX+6,J)=0.0
              W(MX+7,J)=0.0
  670 CONTINUE
  679 CONTINUE
C
C-----INPLANE LOAD DUE TO LATERAL DEFLECTION
C
              EPP=1.0E-5
            DO 680 J=4,MYP4
            DO 680 I=4,MXP4
      E1=   -EX(I,J+1)*AX(I,J+1)*((W(I,J)-W(I-1,J))**2)/(2*HX*HX)
      E2=   -EX(I,J)*AX(I,J)*((W(I,J)-W(I-1,J))**2)/(2*HX*HX)
      E3=   +EX(I+1,J)*AX(I+1,J)*((W(I,J)-W(I+1,J))**2)/(2*HX*HX)
      E4=   +EC(I+1,J+1)*AC(I+1,J+1)*HX*((W(I, J)-W(I+1,J+1))**2)
     5      /(2*HZ**3)
      E5=   -EC(I,J+1)*AC(I,J+1)*HX*((W(I,J)-W(I-1,J+1))**2)/(2*HZ**3)
      E6=   -EC(I,J)*AC(I,J)*HX*((W(I,J)-W(I-1,J-1))**2)/(2*HZ**3)
      E7=   +EC(I+1,J)*AC(I+1,J)*HX*((W(I,J)-W(I+1,J-1))**2)/(2*HZ**3)
            PX2(I,J)=EX(I+1,J+1)*AX(I+1,J+1)*((W(I,J)-W(I+1,J))**2)
     1      /(2*HX*HX)    +E1+E2+E3+E4+E5+E6+E7
      F1=   +EY(I,J+1)*AY(I,J+1)*((W(I,J)-W(I,J+1))**2)/(2*HY*HY)
      F2=   -EY(I,J)*AY(I,J)*((W(I,J)-W(I,J-1))**2)/(2*HY*HY)
      F3=   -EY(I+1,J)*AY(I+1,J)*((W(I,J)-W(I,J-1))**2)/(2*HY*HY)
                PY2(I,J)=EY(I+1,J+1)*AY(I+1,J+1)*((W(I,J)-W(I,J+1))**2)
     1      /(2*HY*HY) +F1+F2+F3
     5 +EC(I+1,J+1)*AC(I+1,J+1)*HY*((W(I,J)-W(I+1,J+1))**2)/(2 *HZ**3)
     6 +EC(I,J+1)*AC(I,J+1)*HY*((W(I,J)-W(I-1,J+1))**2)/(2*HZ**3)
     7 -EC(I,J)*AC(I,J)*HY*((W(I,J)-W(I-1,J-1))**2)/(2*HZ**3)
     8 -EC(I+1,J)*AC(I+1,J)*HY*((W(I,J)-W(I+1,J-1))**2)/(2*HZ**3)
C
C----- ADD INCREMENT OF APPLIED INPLANE LOAD
C
            IF( JIMM .EQ. 1) GO TO 667
      DANGG=KASN/ISYM
      PX2(I,J)=PX2(I,J)+DANGG*XUU(I,J)
      PY2(I,J)=PY2(I,J)+DANGG*XVV(I,J)
  667 CONTINUE
            IF(DABS(PX2(I,J) ) .LT. EPP) PX2(I,J)=0.0
            IF(DABS(PY2(I,J))  .LT.  EPP)  PY2(I,J)=0.0
  680 CONTINUE
  681 CONTINUE
C
C-----CALCULATE INPLANE DISPLACEMENT DUE TO PX2 AND PY2
C
            IF( ITERA .GT. 0) GO TO 691
      CALL DATRUS (NJT,NMEM,HX,HY)
  691 CALL INPLAN (NJT,NMEM)
            DO 685 J=3,MYP5
            DO 685 I=3,MXP5
            IF(DABS(U(I,J)) .LT. EPP) U(I,J)=0.0
            IF(DABS(V(I,J))  .LT.  EPP)  V(I,J)=0.0
            IF(DABS(W(I,J)) .LE. EPP) W(I,J)=0.0
  685 CONTINUE
```

```
C
C     CALCULATE PZZ VERTICAL MEMBRANE FORCE
            SAND=DABS(PZZ(MX/2+4,MY/2+4))
       PZZ IS POSITIVE IN DIRECTION OF QQ1
            DO 700 J=4,MYP4
            DO 700 I=4,MXP4
       G1=   -(2*HX*(U(I,J)-U(I-1,J))+((W(I,J)-W(I-1,J))**2))*(W(I,J)-
     5W(I-1,J))*(EX(I,J)*AX(I,J)+EX(I,J+1)*AX(I,J+1))/ (2*HX**3)
       G2=   +(2*HY*(V(I,J+1)-V(I,J))+((W(I,J+1)-W(I,J))**2))*(W(I,J+1)-
     7      W(I,J))*(EY(I,J+1)*AY(I,J+1)+EY(I+1,J+1)*AY(I+1,J+1))/
     7     (2*HY**3)-(2*HY*(V(I,J)-V(I,J-1))+((W(I,J)-W(I,J-1))**2))*
     8      (W(I,J)-W(I,J-1))*(EY(I+1,J)*AY(I+1,J)+EY(I,J)*AY(I,J))/(2*
     9      HY**3)
            PZZ(I,J)=
     1    (2*HX*(U(I+1,J)-U(I,J))+((W(I+1,J)-W(I,J))**2))
     2    *(W(I+1,J)-W(I,J))*(EX(I+1,J)*AX(I+1,J)+EX(I+1,J+1)*AX(I+1,
     3      J+1))/(2*HX**3) +G1+G2
        GG1=
     1-(HX*(U(I+1,J+1)-U(I,J))+HY*(V(I+1,J+1)-V(I,J))+((W(I,J)-W(I+1,J+1
     2 ))**2)/2)*EC(I+1,J+1)*AC(I+1,J+1)*(W(I,J)-W(I+1,J+1))/( HZ**3)
     3-(HX*(U(I,J)-U(I-1,J+1))+HY*(V(I-1,J+1)-V(I,J))+((W(I,J)-W(I-1,J+1
     4))**2)/2)*AC(I,J+1)*EC(I,J+1)*(W(I,J)-W(I-1,J+1))  /(HZ**3)
       GG2=
     5-(HX*(U(I+1,J-1)-U(I,J))+HY*(V(I,J)-V(I+1,J-1))+((W(I,J)-W(I+1,J-1
     6 ))**2)/2)*AC(I+1,J)*EC(I+1,J)*(W(I,J)-W(I+1,J-1))/(HZ**3)
     7-(HX*(U(I,J)-U(I-1,J-1))+HY*(V(I,J)-V(I-1,J-1))+((W(I,J)-W(I-1,J-1
     8 ))**2)/2)*AC(I,J)*EC(I,J)*(W(I,J)-W(I-1,J-1))/(HZ**3)
          PZZ(I,J)=PZZ(I,J)+  (GG1+GG2)
  700 CONTINUE
            IF(FINDA  EQ  6   AND  FIN  EQ  3 ) GO TO 733
            IF(FINDA .EQ  6  .AND  FIN  NE  3 ) GO TO 978
            BAND=DABS(PZZ(MX/2+4,MY/2+4))
            IF(SAND  GT  BAND) GO TO 24116
C
C-   --NEW LOAD FOR NEXT LOOP
C
C
C        QRESIT   IS POSITIVE IN NEGATIVE DIRECTION OF QQ1
C
C
               TOLE =1 OE-6
            DO 710 J=4,MYP4
            DO 710 I=4,MXP4
  701          QRESIT =QQ1(I,J)-PZZ(I,J)
            QQ2(I,J)=QRESIT
  702          XRESIT=PXX(I,J)
               YRESIT =PYY(I,J)
            IF(DABS(QRESIT) .LT  TOLE ) GO TO 708
            IF(DABS(QQ1(I,J)) .LT  TOLE ) GO TO 705
            IF(DABS(QQ(I,J))  LT  TOLE ) GO TO 704
            QQ1(I,J)=HA*QQ1(I,J)+HN*QQ1(I,J)*QQ(I,J)/QRESIT
            GO TO 707
  704          QQ1(I,J)=QQ1(I,J)- 5*QRESIT
            GO TO 707
  705          QQ1(I,J)=- 3*QRESIT
  707       IF(DABS(QQ1(I,J))  LT  TOLE ) QQ1(I,J)=O O
            GO TO 710
  708       QQ1(I,J)=O O
  710 CONTINUE
            DANO=DABS(QQ2(MX/2+4,MY/2+4))
            IF(DANO  GT  QQ(MX/2+4,MY/2+4)) GO TO 24116
C
C- ---CHECK TO STOP ITERATION
C
            DO 715 J=4,MYP4
            DO 715 I=4,MXP4
            IF(DABS(W(I,J))  GT  WMAX2) WMAX2=DABS(W(I,J))
  715 CONTINUE
               WDIFF=(WMAX1-WMAX2)/(WMAX2)
            IF(DABS(WDIFF) .LT  CLOS) GO TO 730
               ITERA =ITERA +1
            BIGLOD=O.O
            BIGBIG=O.O
            DO 7155 J=4,MYP4
            DO 7155 I=4,MXP4
            IF(DABS(QQ2(I,J)) .GT. BIGLOD ) BIGLOD=QQ2(I,J)
            IF(DABS(QQ(I,J)) .GT. BIGBIG ) BIGBIG=QQ(I,J)
 7155       CONTINUE
            IF(BIGLOD .GT. BIGBIG ) GO TO 730
              WMAX1=WMAX2
              WMAX2=O.O
            IF(ITERA .GE. NITERA ) GO TO 730
            GO TO 401
  730 CONTINUE
            XAY=O.
  732 CONTINUE
C
C-----TOTAL INPLANE DISPLACEMENTS
C
            DO 733 J=4,MYP4
            DO 733 I=4,MXP4
            U(I,J)=U(I,J)+UU(I,J)
            V(I,J)=V(I,J)+VV(I,J)
  733 CONTINUE
C
C     CALCULATE AND PRINT MEMBRANE DIRECT AND SHEAR STRESS
C
            DO 735 J=5,MYP4
            DO 735 I=5,MXP4
        M=I
        MM=J
            K=I-1
            KK=J
        H1=(U(M,MM)-U(K,KK))
        H2=(H1**2)
        H3=((V(M,MM)-V(K,KK))**2)
        H4=((W(M,MM)-W(K,KK))**2)
        H5=(H3**2)
        DSX(I,J)=AX(I,J)*EX(I,J)*HX*(((U(I,J)-U(I-1,J))/HX
     1 +(H2+H3+H4)/(2*HX*HX))/DSQRT ((HX+H1)**2+H3+H4)
     5 +((U(I,J-1)-U(I-1,J-1))/HX+(((U(I,J-1)-U(I-1,J-1))**2)
     6 +((V(I,J-1)-V(I-1,J-1))**2)+((W(I,J-1)-W(I-1,J-1))**2))
     7 /(2*HX*HX))/DSQRT(((HX+U(I,J-1) -U(I-1,J-1))**2)
     8 +((V(I,J-1)-V(I-1,J-1))**2)+((W(I,J-1)-W(I-1,J-1))**2)))
            DSX(I,J)=DSX(I,J)+AC(I,J)*EC(I,J)
     1 *(HX+U(I,J-1)-U(I-1,J))*(HX*(U(I,J-1)-U(I-1,J))
     2 +HY*(V(I-1,J)-V(I,J-1))+0.5*(((U(I,J-1)-U(I-1,J))**2)
     3 +((V(I,J-1)-V(I-1,J))**2)+((W(J,J-1)-W(I-1,J))**2)))
     4/(HZ*HZ*DSQRT(((HX+U(I,J-1)-U(I-1,J))**2)
     5 +((HY+V(I-1,J)-V(I,J-1))**2)+((W(I,J-1)-W(I-1,J))**2)    ))
            DSX(I,J)=DSX(I,J)
     6 +AC(I,J)*EC(I,J)*(HX+U(I,J)-U(I-1,J-1))
     7 *(HX*(U(I,J)-U(I-1,J-1))+HY*(V(I,J)-V(I-1,J-1))
     1 +.5*(((U(I,J)-U(I-1,J-1))**2)+((V(I,J)-V(I-1,J-1))**2)
     9 +((W(I,J)-W(I-1,J-1))**2)))/( HZ*HZ*DSQRT((( HX+U(I,J)
     1 -U(I-1,J-1))**2)+((HY+V(I,J)-V(I-1,J-1))**2)
     2 +((W(I,J)-W(I-1,J-1))**2)))
            DSX(I,J)=DSX(I,J)/(THK*HY)
        M=I-1
        MM=J
        K=I-1
        KK=J-1
        H3=((V(M,MM)-V(K,KK))**2)
        H1=(U(M,MM)-U(K,KK))
        H2=(H1**2)
        H4=((W(M,MM)-W(K,KK))**2)
        H5=(H3**2)
       DSY(I,J)=AY(I,J)*EY(I,J)*HY*(((V(I-1,J)-V(I-1,J-1))/HY
     1 +(H2+H3+H4)/(2*HY*HY))/DSQRT(H2+(HY+H3)**2 +H4)+
     5 ((V(I,J)-V(I,J-1))/HY+(((U(I,J)-U(I,J-1))**2)
     6 +((V(I,J)-V(I,J-1))**2)+((W(I,J)-W(I,J-1))**2))/(2*HY*HY))
     7/DSQRT(((U(I,J)-U(I,J-1))**2)+((HY+V(I,J)-V(I,J-1))**2)
     8 +((W(I,J)-W(I,J-1))**2)))
            DSY(I,J)=DSY(I,J)
     1 +AC(I,J)*EC(I,J)*(HY+V(I-1,J)-V(I,J-1))
     2 *(HX*(U(I,J-1)-U(I-1,J))+HY*(V(I-1,J)-V(I,J-1))
     3 +.5*(((U(I,J-1)-U(I-1,J))**2)+((V(I-1,J)-V(I,J-1))**2)
     4 +((W(I-1,J)-W(I,J-1))**2)))
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```
     5 /(HZ*HZ*DSQRT((( HX+U(I,J-1)-U(I-1,J))**2)
     6 +((HY+V(I-1,J)-V(I,J-1))**2)+((W(I-1,J)-W(I,J-1))**2)))
          DSY(I,J)=DSY(I,J)
     1 +AC(I,J)*EC(I,J)* (HY+V(I,J)-V(I-1,J-1))
     2 *(HX*(U(I,J)-U(I-1,J-1))+HY*(V(I,J)-V(I-1,J-1))
     3 +.5*(((U(I,J)-U(I-1,J-1))**2)+((V(I,J)-V(I-1,J-1))**2)
     4 +((W(I,J)-W(I-1,J-1))**2)))
     5 /(HZ*HZ*DSQRT(((HX+U(I,J)-U(I-1,J-1))**2)
     6 +((HY+V(I,J)-V(I-1,J-1))**2)))
          DSY(I,J)=DSY(I,J)/(THK*HX)
      SHS(I,J)=AC(I,J)*EC(I,J)*((HY+V(I-1,J)-V(I,J-1))
     1 *(HX*(U(I,J-1)-U(I-1,J))+HY*(V(I-1,J)-V(I,J-1))
     2 +.5*(((U(I,J-1)-U(I-1,J))**2)+((V(I,J-1)-V(I-1,J))**2)
     3 +((W(I,J-1)-W(I-1,J))**2)))
     4 /(HZ*HZ*DSQRT(((HX+U(I,J-1)-U(I-1,J))**2)
     5 +((HY+V(I-1,J)-V(I,J-1))**2)+((W(I-1,J)-W(I,J-1))**2))))
          SHS(I,J)=SHS(I,J)-AC(I,J)*EC(I,J)
     6*((HY+V(I,J)-V(I-1,J-1))*(HX*(U(I,J)-U(I-1,J-1))
     7 +HY*(V(I,J)-V(I-1,J-1))+.5*(((U(I,J)-U(I-1,J-1))**2)
     8 +((V(I,J)-V(I-1,J-1))**2)+((W(I,J)-W(I-1,J-1))**2)))
     9 /(HZ*HZ*DSQRT(((HX+U(I,J)-U(I-1,J-1))**2)
     1 +((HY+V(I,J)-V(I-1,J-1))**2)+((W(I,J)-W(I-1,J-1))**2))))
          SHS(I,J)=SHS(I,J)/(THK*HY)
  735 CONTINUE
C
C     CALCULATE BENDING EFFECTS
  738 PRINT 11
      PRINT 13, (AN1(N) ,N=1,40)
      PRINT 16, NPROB,(AN2(N),N=1,35)
      PERCET=KASH
      PERCET=100.*PERCET/ISYM
      PRINT 39,PERCET
          DO 800 J=4,MYP4
          DO 750 I=4,MXP4
            ISTA=I-4
            JSTA=J-4
          DO 740 N=1,3
          K=I+N-2
          DP(N+3)=DSQRT(DX(K,J)*DY(K,J))
          BMX(K,J)=DX(K,J)*(W(K-1,J)-W(K,J) -W(K,J)
     1    +W(K+1,J))/(HX*HX)+DP(N+3)*NNYB(K,J)*(W(K,J-1)
     2    -W(K,J)- W(K,J)+W(K,J+1))/(HY*HY)
          L=J+N-2
          DP(N)=DSQRT(DX(I,L)*DY(I,L))
          BMY(I,L)=DY(I,L)*(W(I,L-1)-2.0*W(I,L)
     1    +W(I,L+1))/(HY*HY)+   DP(N) *NNXB(I,L)
     2    *(W(I-1,L)-2.0*W(I,L)+W(I+1,L))/(HX*HX)
  740 CONTINUE
          QBMX=(BMX(I-1,J)-2.0*BMX(I,J)+BMX(I+1,J))*HY/HX
          QBMY=(BMY(I,J-1)-2.0*BMY(I,J)+BMY(I,J+1))*HX/HY
          QTMX=(W(I-1,J-1)*CX(I,J)-W(I-1,J)*(CX(I,J)
     1    +CX(I,J+1))+W(I-1,J+1)*CX(I,J+1)
     2    -W(I,J-1)*(CX(I,J)+CX(I+1,J))+W(I,J)
     3    *(CX(I,J)+CX(I,J+1)+CX(I+1,J)+CX(I+1,J+1)
     4    )-W(I,J+1)*(CX(I,J+1)+CX(I+1,J+1))
     5    +W(I+1,J-1)*CX(I+1,J)-W(I+1,J)*(CX(I
     6    +1,J)+CX(I+1,J+1))+W(I+1,J+1)*CX(I+1,J
     7    +1))/(HX*HY)
          QTMY=(W(I-1,J-1)*CX(I,J)-W(I-1,J)*(CX(I,J)
     1   +CX(I,J+1))+W(I-1,J+1)*CX(I,J+1)
     2    -W(I,J-1)*(CX(I,J)+CX(I+1,J))+W(I,J)
     3    *(CX(I,J)+CX(I,J+1)+CX(I+1,J)+CX(I+1,J
     4    +1))-W(I,J+1)*(CX(I,J+1)+CX(I+1,J+1))
     5    +W(I+1,J-1)
     5    *CX(I+1,J)-W(I+1,J)*(CX(I+1,J)+CX(I+1,J
     6    +1))+W(I+1,J+1)*CX(I+1,J+1))/(HX*HY)
      QPX=(1.0/HX)*(PX1(I,J)*W(I-1,J)-(PX1(I,J)
     1    +PX1(I+1,J))*W(I,J)+PX1(I+1,J)*W(I+1,J))
      QPY=(1.0/HY)*(PY1(I,J)*W(I,J-1)-(PY1(I,J)
     1    +PY1(I,J+1))*W(I,J)+PY1(I,J+1)*W(I,J+1))
          QPC2=PC2(I+1,J+1)*(W(I+1,J+1)-W(I,J))-PC2(I,J)*(W(I,J)-W(I-1,
     1    J-1))
          QPC2=QPC2/HZ
          QPC1=PC1(I+1,J-1)*(W(I+1,J-1)-W(I,J))-PC1(I,J)*(W(I,J)-W(I-1,
     2    J+1))
          QPC1=QPC1/HZ
      REACT=QBMX+QBMY+QTMX+QTMY
          REACT=REACT-QPX-QPY-QPC1-QPC2
C
C     TOTAL REACTION
C
          IF(FINDA .EQ. 6.) GO TO 748
          REACT=REACT-PZZ(I,J)
  748 CONTINUE
  749 PRINT 45,ISTA,JSTA,W(I,J),U(I,J),V(I,J),REACT
  750 CONTINUE
      PRINT 19
          IF( I .EQ. MX/2+4 .AND. J .EQ. MY/2+4) DREACT=DABS(REACT)
  800 CONTINUE
          IF( FIN .EQ. 3.) PRINT 46, MIZ
           IF(XMART .EQ.  1.) PRINT 46,ITERA
          PERR=(DABS(WDIFF))*100.
          IF( FIN .EQ. 0. .AND. ITERA .GT. NITERA)PRINT 49,PERR
          MIZ=1
          XML=DABS(QQ(MX/2+4,MY/2+4 ) )
          XCHK= 100.*(XML-DREACT)* 1./XML
          XCHK=DABS(XCHK)
          IF(XCHK .GT. 10.0 .AND. XMART .EQ. 1.0 ) GO TO 24116
      PRINT 11
      PRINT 16,NPROB,(AN2(N),N=1,35)
      PRINT 40
          DO 960 J=4,MYP4
          DO 950 I=4,MXP4
            ISTA=I-4
            JSTA=J-4
            TMX=(CX(I,J)+CX(I,J+1)+CX(I+1,J)+CX(I+1,J+1))
     1 *(-.25)*(W(I-1,J-1)-W(I-1,J+1)-W(I+1,J
     2    -1)+W(I+1,J+1))/(4.0*HX*HY)
            TMY=(CX(I,J)+CX(I,J+1)+CX(I+1,J)+CX(I+1,J+1))
     1    *(.25)*(W(I-1,J-1)-W(I-1,J+1)
     2    -W(I+1,J-1)+W(I+1,J+1))/(4.0*HX*HY)
      PRINT 45,ISTA,JSTA,BMX(I,J),BMY(I,J),TMX,TMY
  950 CONTINUE
      PRINT 19
  960 CONTINUE
  961 PRINT 11
      PRINT 16,NPROB,(AN2(N),N=1,35)
      PRINT 41
          DO 975 J=5,MYP4
          DO 970 I=5,MXP4
            ISTA=I-4
            JSTA=J-4
      PRINT 45,ISTA,JSTA,DSX(I,J),DSY(I,J),SHS(I,J)
  970 CONTINUE
      PRINT 19
  975 CONTINUE
          IF(KASH .EQ. ISYM) GO TO 1010
  977 CONTINUE
      PRINT 19
  978 CONTINUE
C
C-----CALCULATE ELEMENTS OF OUT OF PLANE STIFNESS OF THE MEMBRANE
C
          DO 320 J=4,MYP4
          DO 320 I=4,MXP4
          PX(I,J)=(2*HX*(U(I,J)-U(I-1,J))+((W(I,J)-W(I-1,J))**2))*
     1    (EX(I,J)*AX(I,J)+EX(I, J+1)*AX(I,J+1))/(2*HX**3)
          PY(I,J)=(2*HY*(V(I,J)-V(I,J-1))+((W(I,J)-W(I,J-1))**2))*
     1    (EY(I+1,J)*AY(I+1,J)+AY(I,J)*EY(I,J))/(2*HY**3)
          PZ1(I,J)=(HX*(U(I,J)-U(I-1,J+1))+HY*(V(I-1,J+1)-V(I,J))+
     1    ((W(I,J)-W(I-1,J+1))**2)/2)*AC(I,J+1)*EC(I,J+1)/(HZ**3)
          PZ2(I,J)=(HX*(U(I,J)-U(I-1,J-1))+HY*(V(I,J)-V(I-1,J-1))+
     1    ((W(I,J)-W(I-1,J-1))**2)/2)*AC(I,J)*EC(I,J)/(HZ**3)
          Q3(I,J)=QQ1(I,J)
          Q4(I,J)=-PZZ(I,J)
  320 CONTINUE
C
C-----CHECK FOR FORMULATION TO BE USED AND SET THE INCREMENTAL LOADING.
C
          IF(KASH .EQ. ISYM1) GO TO 1010
          IF(FINDA .NE. 6. ) GO TO 23242
          IF(FINDA .EQ. 6. .AND. FIN .NE. 3.) GO TO 24115
23242     CONTINUE
```

```
          DO 24242 J=4,MYP4
          DO 24242 I=4,MXP4
          QQ(I,J)=QQ(I,J)/KASH
24242     CONTINUE
          KASH=KASH+1
          IF(KASH .EQ. ISYM1) GO TO 24116
          DO 24115 J=4,MYP4
          DO 24115 I=4,MXP4
          QQ(I,J)=KASH*QQ(I,J)
24115     CONTINUE
          DO 2007 I=4,MXP4
          DO 2007 J=4,MYP4
            U(I,J)=U(I,J)+UU(I,J)
            V(I,J)=V(I,J)+VV(I,J)
2007  CONTINUE
 1000 CONTINUE
          IF(FINDA .NE. 6.) GO TO 1993
          IF(FINDA .EQ. 6  .AND  FIN .NE. 3.) GO TO 23111
1993      CONTINUE
1666           CONTINUE
C
C------CALCULATION OF NODAL CURVATURES AND THE MIDDLE SURFACE STRAINS
C
          DO 80001 J=4,MYP4
          DO 80001 I=4,MXP4
          IF(S(I,J) .GT. 1.OE+19) GO TO 80001
          PHIX1(I,J)=(W(I-1,J)-2.*W(I,J)+W(I+1,J))/(HX*HX)
          PHIY1(I,J)=(W(I,J-1)-2.*W(I,J)+W(I,J+1))/(HY*HY)
          PHIXY1(I,J)=DABS(W(I+1,J+1)+W(I-1,J-1)-W(I-1,J+1)-
     1    W(I+1,J-1))/(4.*HX*HY)
          XDX=DABS(W(I+1,J)-W(I-1,J))/(2.*HX)
          YDY=DABS(W(I,J+1)-W(I,J-1))/(2.*HY)
          XYDXY=XDX*YDY
          PHIX1(I,J)=PHIX1(I,J)/(DSQRT((1.+XDX**2)**3))
          PHIY1(I,J)=PHIY1(I,J)/(DSQRT((1.+YDY**2)**3))
          PHIXY1(I,J)=2.*PHIXY1(I,J)
80001     CONTINUE
          DO 1001 J=4,MYP4
          DO 1001 I=4,MXP4
          EPSMX1(I,J)=(U(I+1,J)-U(I-1,J))/(2.*HX)+1./(4.*HX*HX)*((
     1    W(I+1,J)-W(I,J))**2+(W(I,J)-W(I-1,J))**2+
     2    (V(I+1,J)-V(I,J))**2+(V(I,J)-V(I-1,J))**2+
     3    (U(I+1,J)-U(I,J))**2+(U(I,J)-U(I-1,J))**2)
          EPSMY1(I,J)=(V(I,J+1)-V(I,J-1))/(2.*HY)+ 1./(4.*HY*HY)*((
     1    W(I,J+1)-W(I,J))**2+(W(I,J)-W(I,J-1))**2+
     2    (V(I,J+1)-V(I,J))**2+(V(I,J)-V(I,J-1))**2+
     3    (U(I,J+1)-U(I,J))**2+(U(I,J)-U(I,J-1))**2)
          EPMXY1(I,J)=(V(I+1,J)-V(I-1,J))/(2.*HX)+(U(I,J+1)-U(I,J-1))/
     1    (2.*HY)+(W(I+1,J)-W(I-1,J))/(2.*HX)*(W(I,J+1)-W(I,J-1))/
     2    (2.*HY)
1001  CONTINUE
C
C- ---EVALUATION OF THE EQUIVALENT MATERIAL PROPERTIES OF THE NODES
C
          MXP3=MX+3
          MYP3=MY+3
          DO 2002 J=5,MYP3
          DO 2002 I=5,MXP3
          IF(PHIX1(I,J) .EQ. 0  .OR. PHIY1(I,J) .EQ. 0 )GO TO 2002
          IF(S(I,J) .GT. 1.OE+19) GO TO 2002
           CALL MATPRP(PHIX1,PHIY1,PHIXY1,EPSMX1,EPSMY1,EPMXY1,EX,EY,
     1 EXS1,EYS1,NNXB,NNYB,NNXS,NNYS,THK,AF,ASF,ASFF,I,J,ALPHA,
     2 YSTAIN,GB,GS,YILSTZ)
2002  CONTINUE
          DO 20005 J=5,MYP3
          EXS1(4,J)=EXS1(5,J)
          EYS1(4,J)=EYS1(5,J)
          NNYB(4,J)=NNYB(5,J)
          NNXB(4,J)=NNXB(5,J)
            GB(4,J)=  GB(5,J)
          GS(4,J)=GS(5,J)
          EX(4,J)=EX(5,J)
          EY(4,J)=EY(5,J)
          NNXS(4,J)=NNXS(5,J)
          NNYS(4,J)=NNYS(5,J)
            EX(MXP4,J)=EX  (MXP3,J)
            EY(MXP4,J)=EY  (MXP3,J)
           EXS1(MXP4,J)=EXS1(MXP3,J)
           EYS1(MXP4,J)=EYS1(MXP3,J)
           NNXB(MXP4,J)=NNXB(MXP3,J)
           NNYB(MXP4,J)=NNYB(MXP3,J)
           NNXS(MXP4,J)=NNXS(MXP3,J)
           NNYS(MXP4,J)=NNYS(MXP3,J)
             GB(MXP4,J)=GB  (MXP3,J)
             GS(MXP4,J)=GS  (MXP3,J)
20005      CONTINUE
           DO 20006 I=4,MXP4
          EXS1(I,4)=EXS1(I,5)
          EYS1(I,4)=EYS1(I,5)
          NNXB(I,4)=NNXB(I,5)
          NNYB(I,4)=NNYB(I,5)
            GB(I,4)=  GB(I,5)
          EX(I,4)=EX(I,5)
          EY(I,4)=EY(I,5)
          NNXS(I,4)=NNXS(I,5)
          NNYS(I,4)=NNYS(I,5)
          GS(I,4)=GS(I,5)
            EX(I,MYP4)=EX   (I,MYP3)
            EY(I,MYP4)=EY   (I,MYP3)
          EXS1(I,MYP4)=EXS1 (I,MYP3)
          EYS1(I,MYP4)=EYS1 (I,MYP3)
          NNXB(I,MYP4)=NNXB (I,MYP3)
          NNYB(I,MYP4)=NNYB (I,MYP3)
          NNXS(I,MYP4)=NNXS (I,MYP3)
          NNYS(I,MYP4)=NNYS (I,MYP3)
            GB(I,MYP4)=GB   (I,MYP3)
          GS  (I,MYP4)= GS  (I,MYP3).
20006     CONTINUE
          XU=HX/(DSQRT(HX*HX+HY*HY))
          XV=HY/(DSQRT(HX*HX+HY*HY))
          DO 1207 J=4,MYP4
          DO 1207 I=4,MXP4
          ST=(XU**4)/EX(I,J)+(1./GS(I,J)-2.*NNXS(I,J)/EX(I,J))*( XU**2)
     1    *(XV**2)+(XV**4)/EY(I,J)
          EC(I,J)=1./ST
1207      CONTINUE
          DO 1209 J=5,MYP4
          DO 1209 I=5,MXP4
          EEX(I,J)=(EX(I,J)+EX(I-1,J)+EX(I,J-1)+EX(I-1,J-1))/4.
          EEY(I,J)=(EY(I,J)+EY(I-1,J)+EY(I,J-1)+EY(I-1,J-1))/4.
          ABX(I,J)=(NNXS(I,J)+NNXS(I-1,J)+NNXS(I,J-1)+NNXS(I-1,J-1))/4.
          ABY(I,J)=(NNYS(I,J)+NNYS(I-1,J)+NNYS(I,J-1)+NNYS(I-1,J-1))/4.
          ABG(I,J)=(EC(I,J)+EC(I-1,J)+EC(I,J-1)+EC(I-1,J-1))/4.
1209      CONTINUE
          DO 1210 J=5,MYP4
          DO 1210 I=5,MXP4
          EC(I,J)=ABG(I,J)
          EX(I,J)=EEX(I,J)
          EY(I,J)=EEY(I,J)
          NNXS(I,J)=ABX(I,J)
          NNYS(I,J)=ABY(I,J)
1210      CONTINUE
C
C-----EVALUATION OF THE MODEL PROPERTIES FOR THE NEWLY CALCULATED
C-----EQUIVALENT MATERIAL PROPERTIES.
C
          DO 1219 J=5,MYP4
          DO 1219 I=5,MXP4
          CX(I,J)=TH**3/24.*(GB(I,J)+GB(I-1,J)+GB(I,J-1)+GB(I-1,J-1))
1219      CONTINUE
          DO 1208 J=5,MYP4
          DO 1208 I=5,MXP4
          AC(I,J)=NNYS(I,J)*THK*EX(I,J)/EC(I,J)*(HZ**3)/(2.*HX*HY*
     1    (1.-NNXS(I,J)*NNYS(I,J)))
          AX(I,J)=THK*(HY*HY-NNYS(I,J)*HX*HX)/(2.*HY*(1.-NNXS(I,J)*
     1    NNYS(I,J)))
          AY(I,J)=THK*(HX*HX-NNXS(I,J)*HY*HY)/(2.*HX*(1.-NNYS(I,J)*
     1    NNXS(I,J)) )
1208      CONTINUE
          DO 1201 J=4,MYP4
          DO 1201 I=4,MXP4
          DX(I,J)=EXS1(I,J)*TH**3/(12.*(1.-NNXB(I,J)*NNYB(I,J))    )
          DY(I,J)=EYS1(I,J)*TH**3/(12.*(1.-NNXB(I,J)*NNYB(I,J))    )
```

```
C
C--    FORM THE SUBMATRICES OF THE COMPOSITE MODEL
C
1201        CONTINUE
            DO 1202 J=4,MYP4
            DO 1202 I=4,MXP4
            CMX=1
            CMY=1
            IF(J  EQ  MYP4) CMY= 5
            IF(J  EQ  4)CMY= 5
            IF(I  EQ  4)CMX= 5
            IF(I  EQ. MXP4) CMX= 5
            CMP=CMX*CMY
            DX(I,J)=CMP*DX(I,J)
            DY(I,J)=CMP*DY(I,J)
1202        CONTINUE
            DO 12020 I=5,MXP3
            DO 12020 J=5,MYP3
            IF(S(I,J)  LT  1 OE+19) GO TO 12020
            CMP= 25
            DX(I,J)=CMP*DX(I,J)
            DY(I,J)=CMP*DY(I,J)
12020       CONTINUE
2311        CONTINUE
            WMAX1=O
            WMAX2=O
            ITERA=O
            DO 1991 J=4,MYP4
            DO 1991 I=4,MXP4
            XWW(I,J)=W(I,J)
1991        CONTINUE
            FIN =O
            IF(FINDA  EQ  6 ) GO TO 24116
            GO TO 401
24116       CONTINUE
            XMART=2
            ICO=ICO+1
            FIN=O
            WMAX2=O
            WMAX1=O
            ITERA=O
            FINDA=6.
            GO TO 1971
1971        CONTINUE
            WMAX2=O
            WMAX1=O
            DO 1941    J=4,MYP4
            DO 1941 I=4,MXP4
            PC1(I,J)=PZ1(I,J)
            PC2(I,J)=PZ2(I,J)
            PX1(I,J)=PX(I,J)
            PY1(I,J)=PY(I J)
1941        CONTINUE
1921        CONTINUE
            DO 7600 J=3,MYP5
            DO 7500 I=3,MXP5
              II=I-2
              AA1(II)=DY(I,J-1)*HXDHY3
              - 25*ODHY2* RY(I,J-1)
            AA2(II,2)=-2 *(ODHXHY*(NNYB(I,J)*DX(I,J)+NNXB(I,J-1)*DY(
     1      I,J-1))
     1        +HXDHY3 *(DY(I,J-1)+DY(I,J)) )
     2        +ODHXHY*(-CX(I,J)-CX(I+1,J)
     3        -CX(I,J)-CX(I+1,J))-ODHY*PY1(I,J)
      IF(II-1)7402,7403,7402
7402  IF(II-MXP3) 7407,7408,7407
7403        AA3(II,3)=HYDHX3*DX(I+1,J)+O 25*ODHX2*RX(I+1,J)
      GO TO 7404
7408        AA3(II,3)=HYDHX3*DX(I-1,J)+.25*ODHX2*RX(I-1,J)
      GO TO 7404
7407        AA3(II,3)=HYDHX3*(DX(I-1,J)+4.O*DX(I,J)
     1        +DX(I+1,J))+HXDHY3*(DY(I,J-1)+4.O
     2      *DY(I,J)+DY(I,J+1))+ODHXHY*4 *(NNYB(I,J)*DX(I,J)+NNXB(I,J)*
     3      DY(I,J))+ODHXHY
     4        *(CX(I,J)+CX(I,J+1)+CX(I+1,J)
     5        +CX(I+1,J+1)+CX(I,J)+CX(I+1,J)
     6        +CX(I,J+1)+CX(I+1,J+1))+ ODHX
     7       *(PX1(I,J)+PX1(I+1,J))+ODHY
     8        *(PY1(I,J)+PY1(I,J+1))+S(I,J)
     9        +.25*ODHX2*(RX(I-1,J)+RX(I+1,J))
     A        +.25*ODHY2*(RY(I,J-1)+RY(I,J+1))
     B        +(PC1(I,J)+PC1(I+1,J-1)+PC2(I,J)+PC2(I+1,J+1))/HZ
 7404 IF(AA3(II,3) .EQ. O.O) AA3(II,3)=1.O
          AA4(II,2) =-2.O*(HXDHY3*(DY(I,J)+DY(I,J+1))
     1      +ODHXHY*(NNYB(I,J)*DX(I,J)+NNXB(I,J+1)*DY(I,J+1)))
     2      +ODHXHY*(-CX(I,J+1)-CX(I+1,J+1)
     3      -CX(I,J+1)-CX(I+1,J+1))-ODHY
     4      *PY1(I,J+1)
          AA5(II)=HXDHY3*DY(I,J+1)
     1        -.25*ODHY2*RY(I,J+1)
            AA6(II)=QQ(I,J)
       IF(II-1) 7410,7410,7405
7405        AA2(II,1)=ODHXHY*(NNYB(I-1,J)*DX(I-1,J)+NNXB(I,J-1)*DY(I,J-1)
     1      )+
     1                 ODHXHY*(CX(I,J)+CX(I,J))
     2                 -PC2(I,J)/HZ
            AA3(II,2)=-2.O*(HYDHX3*(DX(I-1,J)+DX(I,J))
     1      +ODHXHY*(NNYB(I-1,J)*DX(I-1,J)+NNXB(I,J)*DY(I,J)) )
     2        +ODHXHY*(-CX(I,J)-CX(I,J+1)
     3        -CX(I,J)-CX(I,J+1))-ODHX*PX1(I,J)
            AA4(II,1)=ODHXHY*(NNYB(I-1,J)*DX(I-1,J)+NNXB(I,J+1)*DY(I,J+1)
     1      )
     1        +ODHXHY*(CX(I,J+1)+CX(I,J+1))
     2        -PC1(I,J)/HZ
7410  IF(II-2) 7430,7430,7420
 7420       AA3(II,1)= DX(I-1,J)*HYDHX3
     1        -.25*ODHX2*RX(I-1,J)
 7430 IF(II-MXP3) 7440,7450,7450
7440        AA2(II,3)=ODHXHY*(NNYB(I+1,J)*DX(I+1,J)+NNXB(I,J-1)*DY(I,J-1)
     1      )
     1        +ODHXHY*(CX(I+1,J)+CX(I+1,J))
     2        -PC1(I+1,J-1)/HZ
            AA3(II,4)=-2.O*(HYDHX3*(DX(I,J)+DX(I+1,J))
     1      +ODHXHY*(NNYB(I+1,J)*DX(I+1,J)+NNXB(I,J)*DY(I,J))        )
     2        +ODHXHY*(-CX(I+1,J)-CX(I+1,J+1)
     3        -CX(I+1,J)-CX(I+1,J+1))-ODHX
     4        *PX1(I+1,J)
            AA4(II,3)=ODHXHY*(NNYB(I+1,J)*DX(I+1,J)+NNXB(I,J+1)*DY(I,J+1)
     1      )
     1        +ODHXHY*(CX(I+1,J+1)+CX(I+1,J+1))
     2        -PC2(I+1,J+1)/HZ
 7450 IF(II+1-MXP3) 7460,7500,7500
 7460       AA3(II,5)= HYDHX3*DX(I+1,J)
     1        -.25*ODHX2*RX(I+1,J)
7500        CONTINUE
C
C----- BEGIN MAIN SOLUTION FOR COMPOSITE MODEL DEFLECTIONS
C
            DO 1515 I=1,MXP3
            A1(I,1)=A(I,J-1)
            A2(I,1)=A(I,J-2)
            DO 1510 K=1,MXP3
            BB1(I,K)=B(I,K,J-1)
            BB2(I,K)=B(I,K,J-2)
            CC1(I,K)=C(I,K,J-1)
            CC2(I,K)=C(I,K,J-2)
1510        CONTINUE
1515        CONTINUE
            JJ=J
            CALL MATRIX(L1,JJ,MXP3,MY)
            DO 1585 I=1,MXP3
            A(I,J)=AA(I,1)
            DO 1580 K=1,MXP3
            B(I,K,J)=BB(I,K)
            C(I,K,J)=CC(I,K)
1580        CONTINUE
1585        CONTINUE
 7600 CONTINUE
C
C----- COMPUTE LATERAL DEFLECTIONS OF THE COMPOSITE MODEL
C
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      DO 1650 LL=3,MYP5
      J=MY+8-LL
      DO 1625 I=1,MXP3
      W1(I,1)=W(I,J+1)
      W2(I,1)=W(I,J+2)
      AA(I,1)=A(I,J)
      DO 1620 K=1,MXP3
      BB(I,K)=B(I,K,J)
      CC(I,K)=C(I,K,J)
1620  CONTINUE
1625  CONTINUE
      CALL MATMPY(L1,MXP3,1,BB,W1,A1)
      CALL MATMPY(L1,MXP3,1,CC,W2,A2)
      DO 1630 I=1,MXP3
      W(I,J)=AA(I,1)+A1(I,1)+A2(I,1)
1630  CONTINUE
1650  CONTINUE
      W(1,3)=2.*W(1,4)-W(1,5)
      W(MXP3,3)=2.*W(MXP3,4)-W(MXP3,5)
      W(1,MYP5)=2.*W(1,MYP4)-W(1,MYP3)
      W(MXP3,MYP5)=2.*W(MXP3,MYP4)-W(MXP3,MYP3)
      DO 1665 J=3,MYP5
      DO 1660 I=3,MXP5
      II=MXP5+3-I
      W(II,J)=W(II-2,J)
1660  CONTINUE
1665  CONTINUE
      DO 1670 J=3,MYP5
      W(1,J)=0.0
      W(2,J)=0.0
      W(MX+6,J)=0.0
      W(MX+7,J)=0.0
1670  CONTINUE
      IF(ICO .EQ. 1) GO TO 1911
      DO 1910 J=4,MYP4
      DO 1910 I=4,MXP4
      W(I,J)=(W(I,J)+XWW(I,J))/2
1910  CONTINUE
1911  CONTINUE
      ITERA=0
      DEF=0
      DEFL=0
      GO TO 1986
1985  DO 1986 J=4,MYP4
      DO 1986 I=4,MXP4
      IF(LAMB .EQ. 6) QQ(I,J)=QQ(I,J)+( SYM*QQ11(I,J)-QQ(I,J))/2
      IF(LAMB .EQ. 7) QQ(I,J)=SYM*QQ11(I,J)
      IF(LAMB .EQ. 8) STOP
1986  CONTINUE
C
C    CHECK FOR DEF  CONVERGENCE OF THE COMPOSITE MODEL
C
      DO 1997 J=4,MYP4
      DO 1997 I=4,MXP4
      IF(DABS(W(I,J)) .GT. DEF) DEF=W(I,J)
      IF(DABS(XWW(I,J)) .GT. DEFL) DEFL=XWW(I,J)
1997  CONTINUE
      WDI=DEF-DEFL
      IF(DABS(WDI) .GT. ALDF) GO TO 1998
1999  FIN=3
      KASN=KASN+1
      ICO=0
1998  CONTINUE
       MIZ=MIZ+1
      IF(KASH .LE. ISYM1) GO TO 679
9990 CONTINUE
9999 CONTINUE
     STOP
      END
```

```
      SUBROUTINE MATPRP(PHIX1,PHIY1,PHIXY1,EPSMX1,EPSMY1,EPMXY1,EX,EY,
     1  EXS1,EYS1,NNXB,NNYB,NNXS,NNYS,THK,AF,ASF,ASFF,K,L,ALPHA,
     2 YSTAIN,GB,GS,YILSTZ)
      IMPLICIT REAL*8(A-H,O-Z)
C
C
C-----THIS ROUTINE TAKES IN THE CURVATURES AND THE MIDDLE SURFACE
C-----STRAINS OF THE NODES OF THE PLATE AND CALCULATES THE EQUIVALENT
C-----MATERIAL PROPERTIES OF THE NODES OF EACH COMPONENT OF THE PLATE
C-----MODEL.
C
C
      DIMENSION SUB1(21),SUB2(21),SUB3(21)
      DIMENSION PHIX1(27,27),PHIY1(27,27), PHIXY1(27,27),EPSMX1(27,27),
     1 EPSMY1(27,27),EPMXY1(27,27),EXS1(27,27),  EX(27,27),EY(27,27),
     1 EYS1(27,27),NNXB(27,27),NNYB(27,27),NNXS(27,27),NNYS(27,27)
      DIMENSION SIGX(21),SIGY(21),TAU(21),SF(21),SFF(21),F(21),
     1 ASF(27,27,21),ASFF(27,27,21),AF(27,27,21)
      DIMENSION    GB(27,27),GS(27,27)
      DIMENSION EPSX(21),EPSY(21),EPSXY(21)
         REAL*8 NUX,NUY
      REAL*8 NEUO
      REAL*8 NUXS,NUYS,NUXSN,NUYSN,MX,MY,MXY,NUXB,NUYB,NUXBN,NUYBN
            REAL*8 NNXB,NNYB,NNXS,NNYS
            COMMON PR
            COMMON/PRP/XXX
      COMMON/ZAP/ZEPX(27,27,21),ZEPY(27,27,21),ZEPXY(27,27,21),
     1 RAMP(27,27,21)
            YILSTS=YILSTZ
            XM=YILSTZ/YSTAIN
            XXX=YILSTZ
                TH=THK
            I=K
            J=L
              PHIX=PHIX1(I,J)
              PHIY=PHIY1(I,J)
              PHIXY=PHIXY1(I,J)
              EPSMX=EPSMX1(I,J)
              EPSMY=EPSMY1(I,J)
            EPSMXY=DABS(EPMXY1(I,J))
            EPMXY1(I,J)=EPSMXY
            TOLA=.00000001
            IF(EPSMXY .EQ. TOLA .OR. EPSMXY .LE. TOLA) EPSMXY=0.
            IF(PHIXY .EQ. TOLA .OR. PHIXY .LE. TOLA)PHIXY=.0
1320                  FORMAT(2I5)
1321                  FORMAT(6F12.8)
777   DO 121 I=1, 21
       SIGX(I)=0.
       SIGY(I)=0.
       TAU(I)=0.
      SUB1(I)=0.
      SUB2(I)=0.
      SUB3(I)=0.
121   CONTINUE
          NN=21
       NN1=NN-1
       NM=NN1/2+1
       ZINCRT=TH/NN1
       DO 13 I=1,NM
      IM1=I-1
      EPSX(I)=EPSMX+PHIX*IM1*ZINCRT
13    SUB1(I)=EPSX(I)
       DO 131 I=1,NM
       J=IABS(NM+1-I)
131   EPSX(J)=SUB1(I)
       DO 14 I=NM,NN
         JM1=IABS(NM-I)
14     EPSX(I)=EPSMX-PHIX*JM1*ZINCRT
       DO 15 I=1,NM
       IM1=I-1
      EPSY(I)=EPSMY+PHIY*IM1*ZINCRT
15    SUB2(I)=EPSY(I)
       DO 151  I=1,NM
        J=IABS(NM+1-I)
151   EPSY(J)=SUB2(I)
       DO 16 I=NM,NN
       JM1=IABS(NM-I)
   16 EPSY(I)=EPSMY-PHIY*JM1*ZINCRT
```

```
      SS2=SS
      NUYB=NUYB+.0001
30     NUXB=MX/MY*NUYB*PHIY/(PHIX+NUYB*PHIY-MX/MY*NUYB*PHIX)
       EXB=12.*(1.-NUYB*NUXB)/(TH**3.*(PHIX+NUYB*PHIY)) *MX
23     EYB=12.*(1.-NUYB*NUXB )/(TH**3.*(PHIY+NUXB*PHIX))   *MY
24      P=12.*MXY/(TH**3.*PHIXY)
66    FORMAT(7F11.3)
666     SS=DSQRT(EXB*EYB)/(2.* (1.+DSQRT(NUYB*NUXB)))
        SS3=SS1/SS2
            SS4=SS/P
        IF(SS3 .GT. 1. .AND. SS4 .LE. 1.) GO TO 28
        IF(SS3 .LE. 1. .AND. SS4 .GT. 1.) GO TO 28
27       NUYB=NUYB+.0001
        GO TO 30
28       G=P
            IF(NUXB .GT. .5 .OR. NUYB .GT. .5) GO TO 400
            GO TO 10009
10009 CONTINUE
            GB(K,L)=G
            NNXB(K,L)=NUXB
            NNYB(K,L)=NUYB
            EXS1(K,L)=EXB
            EYS1(K,L)=EYB
10034       IF(DABS(EPSMXY)-TOLS)10036,10036,10035
10036       IF(DABS(EPSMX-EPSMY)-TOLL)10079,10079,10078
10079       CONTINUE
            NUXS=NNXS(K-1,L)
            NUYS=NNYS(K-1,L)
            EXS=PX*(1.-NUXS)/(TH*EPSMX)
            EYS=EXS
            G=EXS/(2.*(1.+NUXS))
            IF(NUXS .GT. .5 .OR. NUYS .GT. .5) GO TO 400
            GO TO 10008
10078       CONTINUE
            NUXS=(EPSMX-PX/PY*EPSMY)/(EPSMX*PX/PY-EPSMY)
            NUXS=DABS(NUXS)
            IF(ALPHA .EQ. 0 .AND. NUXS .LE. PR) NUXS=PR
            NUYS=NUXS
        EYS=PY*(1.-NUYS**2.)/(TH*(EPSMY+NUXS*EPSMX))
            EXS=EYS
            G=EYS/(2.*(1.+NUYS))
            IF(NUXS .GT. .5 .OR. NUYS .GT. .5) GO TO 400
         GO TO 10008
10035       CONTINUE
            CONTINUE
            NUYS=.2
         NUXS=PX/PY*NUYS*EPSMY/(EPSMX+NUYS*EPSMY-  PX/PY*NUYS*EPSMX)
            NUXS=DABS(NUXS)
        EXS=PX*(1.-NUYS*NUXS)/(TH*(EPSMX+EPSMY*NUYS))
        EYS=PY*(1.-NUXS*NUYS)/(TH*(EPSMY+EPSMX*NUXS))
         P=PXY/(TH*EPSMXY)
         S=DSQRT(EXS*EYS)/(2.*(1.+DSQRT(NUXS*NUYS))   )
        S1=S
        NUYS=NUYS+.0001
         NUXS=PX/PY*NUYS*EPSMY/(EPSMX+NUYS*EPSMY-  PX/PY*NUYS*EPSMX)
            NUXS=DABS(NUXS)
        EXS=PX*(1.-NUYS*NUXS)/(TH*(EPSMX+EPSMY*NUYS))
        EYS=PY*(1.-NUXS*NUYS)/(TH*(EPSMY+EPSMX*NUXS))
         P=PXY/(TH*EPSMXY)
         S=DSQRT(EXS*EYS)/(2.*(1.+DSQRT(NUXS*NUYS))   )
        S2=S
        NUYS=NUYS+.0001
31       NUXS=PX/PY*NUYS*EPSMY/(EPSMX+NUYS*EPSMY-  PX/PY*NUYS*EPSMX)
            NUXS=DABS(NUXS)
        EXS=PX*(1.-NUYS*NUXS)/(TH*(EPSMX+EPSMY*NUYS))
        EYS=PY*(1.-NUXS*NUYS)/(TH*(EPSMY+EPSMX*NUXS))
         P=PXY/(TH*EPSMXY)
         S=DSQRT(EXS*EYS)/(2.*(1.+DSQRT(NUXS*NUYS))   )
        S4=S/P
        S3=S1/S2
        IF(S3 .LE. 1. .AND . S4 .GT. 1.) GO TO 32
        IF(S3 .GT. 1. .AND. S4 .LE. 1.)  GO TO 32
3004                   FORMAT(F10.4)
33       NUYS=NUYS+.0001
        GO TO 31
32       G=P
            IF(NUXS .GT. .5 .OR. NUYS .GT. .5) GO TO 400
            GO TO 10008
10008 CONTINUE
            EX(K,L)=EXS
            EY(K,L)=EYS
            NNXS(K,L)=NUXS
            NNYS(K,L)=NUYS
            GS(K,L)=G
10020 CONTINUE
            EYS1(K,L)=EYB
            EXS1(K,L)=EXB
            EY(K,L)=EYS
            EX(K,L)=EXS
            NNXB(K,L)=NUXB
            NNYB(K,L)=NUYB
            NNXS(K,L)=NUXS
            NNYS(K,L)=NUYS
 109   FORMAT(2F11.2)
 901  FORMAT(5(5X,F10.4))
 9000 FORMAT(6(2X,F10.4))
            GO TO 26
400         CONTINUE
            EYS1(K,L)=ALPHA
            EXS1(K,L)=ALPHA
            EX(K,L)=ALPHA
            EY(K,L)=ALPHA
            NNXB(K,L)=.5
            NNYB(K,L)=.5
            NNXS(K,L)=.5
            NNYS(K,L)=.5
            GS(K,L)=ALPHA/3.
            GB(K,L)=ALPHA/3.
 26         RETURN
      END
C
C
C
C
      SUBROUTINE FIX1(D,L1,NK)
      IMPLICIT REAL *8(A-H,O-Z)
      DATA L3,L4/-1,-1/
      DIMENSION D(L1,L1),IFIX(100)
            L3=L3*L4
         IF(L3 .LT. 0) GO TO 500
         DO 100 I=1,NK
            IFIX(I)=0
            XX=D(I,I)
         IF(DABS(XX) .LT.  1.0E+15) GO TO 100
         DO 50 L=1,NK
            D(I,L)=0.0
            D(L,I)=0.0
50    CONTINUE
            D(I,I)=1.0
            IFIX(I)=1
100   CONTINUE
      GO TO 900
500   CONTINUE
         DO 600 I=1,NK
         IF(IFIX(I) .EQ. 1) D(I,I)=0.0
600   CONTINUE
900   CONTINUE
      RETURN
      END
```

```
C
C
C   ...
      SUBROUTINE DATRUS(NJT,NMEM,HX,HY)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/DAT/CORDX(441),CORDY(441),CORDZ(441),
     1 EC(27,27),EX(27,27),EY(27,27),AE(1640),JT(1640),KT(1640),
     2  AX(27,27),AY(27,27),AC(27,27)
      COMMON/INCR/MX,MY,MXP2,MXP3,MXP4,MXP5,MXP7,
     1              MYP2,MYP3,MYP4,MYP5,MYP7
C
C-----CONSTANTS
C
              MXP1=MX+1
              MYP1=MY+1
C
C    ... JOINTS CO-ORDINATES
           DO 50 NJ=1,NJT
              CORDX(NJ)=0.0
              CORDY(NJ)=0.0
           CORDZ(NJ)=0.0
   50 CONTINUE

           DO 100 J=1,MYP1
           DO 100 I=1,MXP1
              NJ=(J-1)*MXP1+I
              I1=I+3
              J1=J+3
              CORDX(NJ)=(I-1)*HX
              CORDY(NJ)=(J-1)*HY
  100 CONTINUE
C
C   --CALCULATE MEMBER CONNECTIVITY AND ITS STIFFNESS
C

C   ---MEMBER IN X-DIRECTION
              MN=0
           DO 330 JJ=1,MYP1
           DO 330 II=1,MX
              I1=II+4
           J1=JJ+4
              MN=MN+1
              JT(MN)=(JJ-1)*MXP1+II
              KT(MN)=(JJ-1)*MXP1+(II+1)
      IF(JJ .NE. 1) GO TO 310
              AE(MN)=AX(I1,J1)*EX(I1,J1)
           GO TO 330
  310 IF(JJ .NE. MYP1) GO TO 320
              AE(MN)=AX(I1,J1-1)*EX(I1,J1-1)
      GO TO 330
  320         AE(MN)=AX(I1,J1)*EX(I1,J1)+AX(I1,J1-1)*EX(I1,J1-1)
  330 CONTINUE
C
C-----MEMBER IN Y-DIRECTION
C
           DO 370 II=1,MXP1
           DO 370 JJ=1,MY
              I1=II+4
              J1=JJ+4
              MN=MN+1
              JT(MN)=(JJ-1)*MXP1+II
              KT(MN)=(JJ)*MXP1+II
      IF(II .NE. 1) GO TO 360
              AE(MN)= AY(I1,J1)*EY(I1,J1)
           GO TO 370
  360 IF(II .NE. MXP1) GO TO 365
              AE(MN)=AY(I1-1,J1)*EY(I1-1,J1)
           GO TO 370
  365         AE(MN)=AY(I1,J1)*EY(I1,J1)+AY(I1-1,J1)*EY(I1-1,J1)
  370 CONTINUE
C
C-----MEMBER IN XY DIRECTION
C
           DO 380 JJ=1,MY
           DO 380 II=1,MX
              I1=II+4
              J1=JJ+4
              MN=MN+1
              JT(MN)=(JJ-1)*MXP1+II
              KT(MN)=(JJ)*MXP1+(II+1)
              AE(MN)=AC(I1,J1)*EC(I1,J1)
  380 CONTINUE
           DO 390 JJ=2,MYP1
           DO 390 II=1,MX
              I1=II+4
              J1=JJ+4
              MN=MN+1
              JT(MN)=(JJ-1)*MXP1+II
              KT(MN)=(JJ-2)*MXP1+(II+1)
              AE(MN)=AC(I1,J1-1)*EC(I1,J1-1)
  390 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE MATRIX(L1,JJ,MXP3,MY)
      IMPLICIT REAL *8(A-H,O-Z)
      COMMON/MATR/ AA1(23),AA2(23,3),AA3(23,5),AA4(23,3),
     1 AA5(23),AA6(23),AA(23,1),A1(23,1),A2(23,1),
     2 BB(23,23),BB1(23,23),BB2(23,23),CC(23,23),
     3 CC1(23,23),CC2(23,23),AAUG(23,23,2),D(23,23),E(23,23),D1(23,23)
      CALL MATMY1(L1,MXP3,MXP3,AA1,BB2,E)
      CALL MATA1(L1,MXP3,E,AA2,E)
      CALL MATMPY(L1,MXP3,MXP3,E,BB1,D)
      CALL MATMY1(L1,MXP3,MXP3,AA1,CC2,CC)
           DO 535 K=1,MXP3
           DO 530 I=1,MXP3
             D(I,K)=D(I,K)+CC(I,K)
  530 CONTINUE
  535 CONTINUE
      CALL MATA2(L1,MXP3,D,AA3,D)
      CALL INVR6(D,L1,MXP3)
           DO 545 K=1,MXP3
           DO 540 I=1,MXP3
             D(I,K)=-D(I,K)
  540 CONTINUE
  545 CONTINUE
      CALL MATM2(L1,MXP3,D,AA5,CC)
      CALL MATMPY(L1,MXP3,MXP3,E,CC1,BB)
      CALL MATA1(L1,MXP3,BB,AA4,BB1)
      CALL MATMPY(L1,MXP3,MXP3,D,BB1,BB)
      CALL MATMPY(L1,MXP3,1,E,A1,AA)
      CALL MATMY1(L1,MXP3,1,AA1,A2,A1)
           DO 560 I=1,MXP3
             A2(I,1)=AA(I,1)+A1(I,1)
  560 CONTINUE
           DO 570 I=1,MXP3
             A1(I,1)=A2(I,1)-AA6(I)
  570 CONTINUE
      CALL MATMPY(L1,MXP3, 1,D,A1,AA)
      RETURN
      END
C
C
C
C
      SUBROUTINE MATMPY(M1,L2,L,X,Y,Z)
      IMPLICIT REAL *8(A-H,O-Z)
      DIMENSION X(M1,M1),Y(M1,L),Z(M1,L)
           DO 100 I=1,L2
           DO 100 M=1,L
             Z(I,M)=0.0
           DO 100 K=1,L2
             Z(I,M)=X(I,K)*Y(K,M)+Z(I,M)
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATM2(M1,L2,Y,X,Z)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(M1),Z(M1,M1),Y(M1,M1)
            DO 100 I=1,L2
            DO 100 J=1,L2
              Z(I,J)=X ( J)*Y(I,J)
  100 CONTINUE
      RETURN
      END


      SUBROUTINE MATA1(M1,L2,Z,X3,Y)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X3(M1,3),Z(M1,M1),Y(M1,M1)
            MM1=L2-1
            DO 50 I=1,L2
            DO 50 J=1,L2
              Y(I,J)=Z(I,J)
   50 CONTINUE
            DO 100 I=2,MM1
              Y(I,I-1)=Y(I,I-1)+X3(I,1)
              Y(I,I)=Y(I,I)+X3(I,2)
              Y(I,I+1)=Y(I,I+1)+X3(I,3)
  100 CONTINUE
              Y(1,1)=Y(1,1)+X3(1,2)
              Y(1,2)=Y(1,2)+X3(1,3)
              Y(L2,L2-1)=Y(L2,L2-1)+X3(L2,1)
              Y(L2,L2)=Y(L2,L2)+X3(L2,2)
      RETURN
      END
       SUBROUTINE MATA2(L1, M1, Z,X3,Y)
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION X3(L1,5),Z(L1,L1),Y(L1,L1)
            MM2=M1-2
          DO 100 I=3,MM2
            Y(I,I-2)=Z(I,I-2)+X3(I,1)
            Y(I,I-1)=Z(I,I-1)+X3(I,2)
            Y(I,I)=Z(I,I)+X3(I,3)
            Y(I,I+1)=Z(I,I+1)+X3(I,4)
            Y(I,I+2)=Z(I,I+2)+X3(I,5)
  100 CONTINUE
            Y(1,1)=Z(1,1)+X3(1,3)
            Y(1,2)=Z(1,2)+X3(1,4)
            Y(1,3)=Z(1,3)+X3(1,5)
            Y(2,1)=Z(2,1)+X3(2,2)
            Y(2,2)=Z(2,2)+X3(2,3)
            Y(2,3)=Z(2,3)+X3(2,4)
            Y(2,4)=Z(2,4)+X3(2,5)
            Y(M1-1,M1-3)=Z(M1-1,M1-3)+X3(M1-1,1)
            Y(M1-1,M1-2)=Z(M1-1,M1-2)+X3(M1-1,2)
            Y(M1-1,M1-1)=Z(M1-1,M1-1)+X3(M1-1,3)
            Y(M1-1,M1)=Z(M1-1,M1)+X3(M1-1,4)
            Y(M1,M1-2)=Z(M1,M1-2)+X3(M1,1)
            Y(M1,M1-1)=Z(M1,M1-1)+X3(M1,2)
            Y(M1,M1)=Z(M1,M1)+X3(M1,3)
       RETURN
       END


      SUBROUTINE MATMY1(M1,L2,L,X,Y,Z)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(M1),Z(M1,L),Y(M1,L)
            DO 100 I=1,L2
            DO 100 J=1,L
            Z(I,J)=X(I)*Y(I,J)
  100 CONTINUE
      RETURN
      END
```

```
C
C
      SUBROUTINE INTERP (I1,J1,I2,J2,D,ICARD,Z,IS,IB,IG,L2,L3,ICX,ICY)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION I1(20),I2(20),J1(20),J2(20),D(20),Z(27,27)
      COMMON/INCR/MX,MY,MXP2,MXP3,MXP4,MXP5,MXP7,
     1              MYP2,MYP3,MYP4,MYP5,MYP7
   10 FORMAT(/,'DATA TYPE NOT PROPERLY DEFINED FOR INTERP ')
   20 FORMAT(/, ' ERROR IN INPUT OF DATA FOR DISTRIBUTION ')
   30 FORMAT(/51H   STATIONS NOT IN PROPER ORDER FOR INTERPOLATION   )
            EP=1.0E-15
C
C-----ZERO STORAGE BLOCK
C
            DO 100 J=1,MYP7
            DO 50 I=1,MXP7
              Z(I,J)=0.0
   50 CONTINUE
  100 CONTINUE
      IF(ICARD .EQ. 0) GO TO 3000
              IPL=4
            DO 2000 L=1,ICARD
              IX1=I1(L)+IPL
              IX2=I2(L)+IPL
              JY1=J1(L)+IPL
              JY2=J2(L)+IPL
      IF(IX2 .LT. IX1) GO TO 5500
      IF(JY2 .LT. JY1) GO TO 5500
              ISW=0
              JSW=0
      IF(IX2 .GT. IX1) ISW=1
      IF(JY2 .GT. JY1) JSW=1
C
C
C------DISTRIBUTE DATA OVER AREA DEFINED BY IX1,JX1,IX2,JX2
C
C-----CHECK FOR TYPE OF DATA
      IF(IS .GT. 0) GO TO 700
      IF(IB .GT. 0)GO TO 200
      IF(IG .GT. 0) GO TO 300
C
C
C-----TYPE OF DATA DEFINED---ERROR
      PRINT 10
            GO TO 6000
C-----SET UP INTERPOLATION FOR GRID AND BAR TYPE DATA
  200 IF(ICX.EQ. 1) GO TO 250
      IF(JSW .GT. 0) GO TO 500
      IF(D(L) .EQ. 0.0) GO TO 2000
            GO TO 275
  250 IF(ISW .GT. 0) GO TO 500
      IF(D(L) .EQ. 0.0) GO TO 2000
  275 PRINT 10
            GO TO 6000
  300 IF(ISW .EQ. 1) GO TO 400
      IF(D(L) .EQ. 0.0) GO TO 2000
      PRINT 20
            GO TO 6000
  400 IF(JSW .EQ. 1) GO TO 450
      IF(D(L) .EQ. 0.0) GO TO 2000
      PRINT 20
            GO TO 6000
  450         IX1=IX1+1
              JY1=JY1+1
  500 IF(ICX .EQ. 1) IX1=IX1+1
      IF(ICY .EQ. 1) JY1=JY1+1
              ISW=0
            JSW=0
  700       DO 1600 J=JY1,JY2
            DO 1500 I=IX1,IX2
```

```
            CMX=1.0
            CMY=1.0
      IF(ISW .EQ. 0) GO TO 900
      IF(JSW  .EQ. 0) GO TO 800
      IF(J .EQ. JY1) CMY=0.5
      IF(J.EQ. JY2) CMY=.5
      IF(I .EQ. IX1) CMX=.5
      IF(I .EQ. IX2) CMX=.5
           GO TO 1000
  800 IF(I  EQ. IX1) CMX=.5
      IF(I .EQ. IX2) CMX=.5
           GO TO 1000
  900 IF(JSW .EQ. 0) GO TO 1000
      IF(J .EQ. JY1) CMY=.5
      IF(J .EQ. JY2) CMY=.5
 1000       CMP=CMX*CMY
      IF(DABS(D(L)) .LT. EP) D(L)=0.0
      IF(DABS(Z(I,J)) .LT. EP) Z(I,J)=.0
            Z(I,J)=Z(I,J)+CMP*D(L)
 1500 CONTINUE
 1600 CONTINUE
 2000 CONTINUE
 3000 RETURN
 5500 PRINT 30
 6000 CONTINUE
       RETURN
      STOP


           END
C
C
C
C
C
      SUBROUTINE MLTXL(X,L1,L2)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(L1,L1)
            DO 200 I=1,L2
            DO 150 J=1,I
              SUM=0.0
            DO 100 K=I,L2
              SUM=SUM+X(K,I)*X(K,J)
  100 CONTINUE
              X(I,J)=SUM
  150 CONTINUE
  200 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE INVLT1(X,L1,L2)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(L1,L1)
            DO 50 I=1,L2
              X(I,I)=1.0/X(I,I)
   50 CONTINUE
              L2M1=L2-1
            DO 200 J=1,L2M1
              JP1=J+1
            DO 150 I=JP1,L2
              IM1=I-1
              SUM=0.0
            DO 120 K=J,IM1
              SUM=SUM-X(I,K)*X(K,J)
  120 CONTINUE
              X(I,J)=X(I,I)*SUM
  150 CONTINUE
  200 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RTSR(R,S,X)
C
C---- FORM PRODUCT (RT*S*R=X)
      IMPLICIT REAL*8(A-H,O-Z)
      DATA ZERO/0.0000/
      DIMENSION R(6,6),S(6,6), X(6,6),T(6,6)
            DO 20 I=1,6
            DO 20 J=1,6
             TEMP=ZERO
            DO 10 K=1,6
             TEMP=TEMP+R(K,I)*S(K,J)
   10 CONTINUE
             T(I,J)=TEMP
   20 CONTINUE
            DO 40 I=1,6
             DO 40 J=1,6
             TEMP=ZERO
            DO 30 K=1,6
             TEMP=TEMP+T(I,K)*R(K,J)
   30 CONTINUE
             X(I,J)=TEMP
   40 CONTINUE
      RETURN
      END
C
C
C
C
      SUBROUTINE INVR6(X,L1,L2)
      IMPLICIT REAL *8(A-H,O-Z)
      REAL*8 ABSF
C
C     THIS ROUTINE TAKES THE INVERSE OF A SYMMETRIC POSITIVEDEF.
C     MATRIX USING A COMPACTED CHOLESKI DECOMPOSITION PROCEDURE .
C     A FULL DIMENSIONED MATRIX IS REQUIRED BUT ONLY THE LOWER
C     HALF IS USED BY THE THREE ROUTINES DRIVEN BY INVR6
      DIMENSION X(L1,L1)
            IF(L2-1)600,10,20
   10       IF(DABS(X(1,1)) .LT. E-10) GO TO 600
              X(1,1)=1.0/X(1,1)
            GO TO 500
   20       IF(L2-2) 30,30,40
   30         S1=X(1,1)*X(2,2)-X(1,2)*X(2,1)
            IF (DABS(S1) .LT. E-10) GO TO 600
              S1=1./S1
              S=X(1,1)
              X(1,1)=S1*X(2,2)
            X(2,2)=S1*S
            X(1,2)=-S1*X(1,2)
            X(2,1)=-S1*X(2,1)
      GO TO 500
   40 CALLFIX1(X,L1,L2)
      CALL DCOM1(X,L1,L2)
      CALL INVLT1(X,L1,L2)
      CALL MLTXL(X,L1,L2)
      CALL FIX1(X,L1,L2)
            DO 100 I=2,L2
              KC=I-1
            DO 50 J=1,KC
              X(J,I)=X(I,J)
   50 CONTINUE
  100 CONTINUE
  500 RETURN
  600 PRINT 601,((X(I,J),J=1,L2),I=1,L2)
  601 FORMAT(1H1,30H SINGULAR MATRIX ENCOUNTERED ./,2(5X,2E15.7))
      STOP
      END
```

```
C
C
      SUBROUTINE INPLAN(NJT,NMEM)
      IMPLICIT REAL*8(A-H,O-Z)
C
C---THIS SUBROUTINE CALCULATES THE RESULTANT FORCES AT EACH JOINT
C-      IN THE MEMBRANE MODEL
C
      DIMENSION SM(6,6),RT(6,6),SSU(441),SSV(441),F(1323),DF(1323),
     1 U(1323),FX(441),FY(441),RZ(441)
      COMMON/STIFF/SI(1323,69)
      COMMON/DAT/CORDX(441),CORDY(441),CORDZ(441),
     1 EC(27,27),EX(27,27),EY(27,27),AE(1640),JT(1640),KT(1640),
     2  AX(27,27),AY(27,27),AC(27,27)
      COMMON/PLANE/PX (27,27),PY (27,27),W(27,27),SU(27,27),SV(27,27),
     1 UU(27,27),VV(27,27)
      COMMON/INCR/MX,MY,MXP2,MXP3,MXP4,MXP5,MXP7,
     1                MYP2,MYP3,MYP4,MYP5,MYP7

C---   FORMAT STATEMENTS
C
 2090 FORMAT(8X,I5,7X,1PD10.3,5X,D10.3,5X,D10.3)
22300 FORMAT(//45H          JT.NO         X-DISP.        Y-DISP
     1   15H    Z-DISP.       /)
 2270 FORMAT(// 28H          MEM. AXIAL FORCE     /)
 2280 FORMAT(8X,I5,2X, 1P3D11.3,3X,3D11.3)
22900 FORMAT(///38H   SUPPORT REACTIONS .LOADING NO.           ,I5,
     1  //  45H      JT.NO.     X-REACT       Y-REACT.
     2    15H      Z-REACT.        ./)
      REAL*8 SORT
            SORT(X)=DSORT(X)
            MXP2=MX+2
            MYP2=MY+2
            MXP1=MX+1
      NSIZE=1323
      NWIDE=69
         DO 100 I=4,MXP4
         DO 100 J=4,MYP4
            NJ=(J-4)*MXP1+(I-3)
            FX(NJ)=PX(I,J)
            FY(NJ)=PY(I,J)
            SSU(NJ)=SU(I,J)
            SSV(NJ)=SV(I,J)
            RZ(NJ)=W(I,J)
            CORDZ(NJ)=W(I,J)
  100 CONTINUE

C---   INITIALIZE ARRAYS
            NOF=3*NJT
            MWD=3*MXP2+3
         DO 270 L=1,NOF
            U(L)=0.0
            DF(L)=0.0
         DO 260 K=1,MWD
            SI(L,K)=0.0
  260 CONTINUE
  270 CONTINUE
C
C      SET UP STRUCTURE STIFFNESS MATRIX
         DO 330 MN=1,NMEM
            JMN=JT(MN)
            KMN=KT(MN)
            DX=CORDX(KMN)-CORDX(JMN)
            DY=CORDY(KMN)-CORDY(JMN)
            DZ=CORDZ(KMN)-CORDZ(JMN)
            XL=SORT(DX*DX+DY*DY+DZ*DZ)
            CX=DX/XL
            CY=DY/XL
            CZ=DZ/XL
            AEM=AE(MN)
      CALL TRSTF(CX,CY,CZ,XL,AEM,SM,RT)
  310 CALL RTSR(RT,SM,SM)
C
C-----ADD MEMBER STIFFNESS MATRIX TO STRUCTUE STIFFNESS MATRIX
            IROW1=3*(JT(MN)-1)
            IROW2=3*(KT(MN)-1)
            JSTRT=1
            JSTOP=3
         DO 330 I=1,3
         DO 320 JS=JSTRT,JSTOP
            SI(IROW1+I,JS)=SI(IROW1+I,JS)+SM(I,JS+I-1)
            SI(IROW2+I,JS)=SI(IROW2+I,JS)+SM(I+3,JS+I+2)
  320 CONTINUE
            JSTOP=JSTOP-1
      IF(JT(MN) .GT. KT(MN)) GO TO 324
            IS=IROW2-IROW1+1
         DO 322 JS=1,3
            SI(IROW1+I,JS+IS-I)=SM(I,JS+3)
  322 CONTINUE
         GO TO 330
  324       IS=IROW1-IROW2+1
         DO 326 JS=1,3
            SI(IROW2+I,JS+IS-I)=SM(I+3,JS)
  326 CONTINUE
  330 CONTINUE
C
C----ADD ELASTIC RESTRAINTS AND REVISE FOR SPECIFIED DISPLACEMENTS
C------ ADD ELASTIC RESTRAINTS TO STIFFNESS MATRIX
C------ SET UP TEMPORARY LOAD VECTOR TO ACCOUNT FOR SPECIFIED DISPLACEMENT
         DO 410 I=1,NJT
            JN=3*(I-1)
            JROW=JN+3
            SI(JN+1,1)=SI(JN+1,1)+SSU(I)
            SI(JN+2,1)=SI(JN+2,1)+SSV(I)
            SI(JN+3,1)=SI(JN+3,1)+1.00+50
  410 CONTINUE
C
C-----  DECOMPOSED STIFFNESS MATRIX
      CALL DCMPBD(SI,NWIDE,NSIZE,MWD,NOF)
C-------INITIALIZE LOAD VECTOR
  420    DO 430 I=1,NOF
            F(I)=0.0
  430 CONTINUE
C-----READ AND ECHO JOINT LOADS
         DO 440 I=1,NJT
            JROW=3*(I-1)
            F(JROW+1)=FX(I)+DF(JROW+1)
         F(JROW+2)=FY(I)+DF(JROW+2)
            F(JROW+3)=DF(JROW+3)
  440 CONTINUE
C
C------
C----- SOLVE FOR JOINT DISPLACEMENTS
      CALL SLVBD(SI,F,U,NWIDE,NSIZE,MWD,NOF)
         DO 510 J=4,MYP4
         DO 510 I=4,MXP4
            NJ=(J-4)*MXP1+(I-3)
            JN=3*(NJ-1)
            UU(I,J)=U(JN+1)
            VV(I,J)=U(JN+2)
  510 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE DCOM1(X,L1,L2)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(L1,L1),T(100)
   10 FORMAT( /85X,'NON-POSITIVE DEFINITE MATRIX ENCOUNTERED')
   15 FORMAT(/,5X,13E10.3)
         DO 20 I=1,L2
            T(I)=X(I,I)
   20 CONTINUE
         IF(X(1,1) .LE. 0.0 ) GO TO 4000
            X(1,1)=DSORT(X(1,1))
            S1=1./X(1,1)
         DO 50 I=2,L2
            X(I,1)=X(I,1)*S1
```

```
 50 CONTINUE
          L2M1=L2-1
        DO 200 J=2,L2M1
          S=0.0
          JM1=J-1
        DO 120 K=1,JM1
        S=S+X(J,K)*X(J,K)
120 CONTINUE
        IF(X(J,J).LE. S) GO TO 4000
          X(J,J)=DSORT(X(J,J)-S)
          S1=1.0/X(J,J)
          JP1=J+1
        DO 190 I=JP1,L2
          S=0.0
        DO 180 K=1,JM1
          S=S+X(I,K)*X(J,K)
180 CONTINUE
          X(I,J)=(X(I,J)-S)*S1
190 CONTINUE
200 CONTINUE
          S=0.0
        DO 250 K=1,L2M1
          S=S+X(L2,K)*X(L2,K)
250 CONTINUE
          S=X(L2,L2)-S
        IF(S .LE. 0.0) GO TO 4000
          X(L2,L2)=DSORT(S)
    RETURN
4000 PRINT 10
          X(1,1)=T(1)
        DO 400 I=2,L2
          K=I-1
          X(I,I)=T(I)
        DO 350 J=1,K
          X(I,J)=X(J,I)
350 CONTINUE
400 CONTINUE
        DO 500 I=1,L2
500 PRINT 15,(X(I,J),J=1,L2)
    STOP
    END


      SUBROUTINE SLVBD(S,F,U,NWIDE,NSIZE,MWD,NOF      )

      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION S(NSIZE,NWIDE),F(NSIZE),U(NSIZE)
            DO 120 I=1,NOF
              J=I-MWD+1
            IF(I+1 .LE. MWD) J=1
              SUM=F(I)
      IF(I  EQ. 1) GO TO 110
              JLIM=I-1
            DO 100 K=J,JLIM
              SUM=SUM-S(K,I-K+1)*U(K)
  100 CONTINUE
  110         U(I)=SUM*S(I,1)
  120 CONTINUE
            DO 150 II=1,NOF
              I=NOF+1-II
              J=I+MWD-1
      IF(J  GT. NOF) J=NOF
              SUM=U(I)
      IF(I+1 .GT. J) GO TO 140
            KS=I+1
            DO 130 K=KS,J
              SUM=SUM-S(I,K-I+1) *U(K)
  130 CONTINUE
  140         U(I)=SUM*S(I,1)
  150 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TRSTF(CX,CY,CZ,XL,AE,S,R)
C
C
C----- SET UP STIFFNESS AND TRANSFORMATION MATRICES FOR SPACE TRUSS
      IMPLICIT REAL *8(A-H,O-Z)
      DIMENSION S(6,6),R(6,6)
      DATA ZERO, EP/0.0000, 1.00-06/,ONE/1.0000/
            DO 100 I=1,6
            DO 100 J=1,6
              S(I,J)=ZERO
              R(I,J)=ZERO
  100 CONTINUE
              AEOL=AE/XL
              S(1,1)=AEOL
              S(1,4)=-AEOL
              S(4,1)=-AEOL
              S(4,4)=AEOL
              D=DSORT(CX*CX+CZ*CZ)
      IF(D.LE. EP) GO TO 120
              R(1,1)=CX
              R(1,2)=CY
              R(1,3)=CZ
              R(2,1)=-CX*CZ/D
              R(2,2)=D
              R(2,3)=-CY*CZ/D
              R(3,1)=-CZ/D
              R(3,3)=CX/D
            DO 110 I=1,3
            DO 110 J=1,3
              R(I+3,J+3)=R(I,J)
  110 CONTINUE
      RETURN
  120          R(1,2)=CY
              R(2,1)=-CY
            R(3,3)=ONE
              R(4,5)=CY
              R(5,4)=-CY
              R(6,6)=ONE
      RETURN
      END
C
C
      SUBROUTINE DCMPBD(S,NWIDE,NSIZE,MWD,NOF)
C
C----- DECOMPOSE BANDED STIFFNESS MATRIX
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION S(NSIZE,NWIDE)
              EP=1.0E-6
            DO 120 I=1,NOF
              II=NOF-I+1
      IF(MWD .LT. II) II=MWD
            DO 120 J=1,II
              IJ=MWD-J
      IF(I-1 .LT. IJ) IJ=I-1
      IF(DABS(S(I,J)) .LT. EP) S(I,J)=0.0
              SUM=S(I,J)
      IF(IJ .LT. 1) GO TO 102
            DO 100 K=1,IJ
      IF(DABS(S(I-K,K+1)) .LT. EP) S(I-K,K+1) =0.0
      IF(DABS(S(I-K,K+J)) .LT. EP) S(I-K,K+J)=0.0
      IF(S(I-K,K+1) .EQ. 0.0 .OR. S(I-K,K+J) .EQ. 0.0) GO TO 100
              SUM=SUM-S(I-K,K+1)*S(I-K,K+J)
  100 CONTINUE
  102 IF(J .NE. 1) GO TO 110
              TEMP=1.0/DSORT(SUM)
              S(I,J)=TEMP
            GO TO 120
  110         S(I,J)=SUM*TEMP
  120 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SUB(AF,ASF,ASFF,II,JJ,THK,EPSX,EPSY,EPSXY,ALPHA,YSTAIN,
     1  SIGX,SIGY,TAU ,KH)
       IMPLICIT REAL *8(A-H,O-Z)
C
C
C-----THIS ROUTINE TAKES IN THE STATE OF STRAIN AT A POINT AT ANY
C-----LAYER OF THE PLATE AND CALCULATES THE STATE OF STRESS AT THAT
C-----POINT
C
C
      DIMENSION AF(27,27,21),ASF(27,27,21),ASFF(27,27,21)
      DIMENSION SIGX(21),SIGY(21),  TAU(21),EPSX(21),EPSY(21),EPSXY(21),
     1 F(21),SF(21),SFF(21) ,EPPX(21),EPPY(21),EPPXY(21)
           REAL*8 NEUO,NUXB1,NUYB1,NUXS1,NUYS1
           COMMON PR
           COMMON/PRP/XXX
      COMMON/ZAP/ZEPX(27,27,21),ZEPY(27,27,21),ZEPXY(27,27,21),
     1 RAMP(27,27,21)
           YILSTZ=XXX
             I=1
   24        YILSTS=AF(II,JJ,I)
           XKAJ=1.
           KAR=0
           ARD=1000000.
             NEUO=ASFF(II,JJ,I)
             EO=ASF(II,JJ,I)
             GO=ASF(II,JJ,I)/(2.*(1.+NEUO))
         EPSXY(I)=.5*EPSXY(I)
           EPPX(I)=O.
           EPPY(I)=O.
           EPPXY(I)=O.
      DEPX=O.
      DEPY=O.
      DEPXY=O.
           TH=THK
           IF(RAMP(II,JJ,I) .NE. O.) GO TO 1025
 90   SIGX(I)=EO/(1.-NEUO**2.)*(EPSX(I)-EPPX(I)-DEPX+NEUO*(EPSY(I)-
     2 EPPY(I)-DEPY))
      SIGY(I)=EO/(1.-NEUO**2.)*(EPSY(I)-EPPY(I)-DEPY+NEUO*(EPSX(I)-
     2  EPPX(I)-DEPX))
      TAU(I)=2.*GO*(EPSXY(I)-EPPXY(I)-DEPXY)
      SIGEFF=DSQRT(SIGX(I)*SIGX(I)+SIGY(I)*SIGY(I)+3.*TAU(I)*TAU(I)-
     2  SIGX(I)*SIGY(I))


           DAN=SIGEFF-YILSTS
           RAN=YILSTZ/YSTAIN
           IF(ALPHA .GT. O.) GO TO 131
           IF(DAN .LT. O. .AND. EO .LT. RAN) GO TO 1025
           IF(SIGEFF-YILSTS)17,181,181
  131      CONTINUE
           IF(DAN .LT. O. .AND. EO .LT. RAN) GO TO 1025
           IF(SIGEFF-YILSTS) 17,17,181
 17    DELEPP=O.
         GO TO 23
1025       KH=KH+1
           DELEPP=O.
           AJ=3.
           BJ=DSQRT(AJ)
           SIGX(I)=RAN/(1.-RR**2)*(EPSX(I)-ZEPX(II,JJ,I))
           SIGY(I)=RAN/(1.-PR**2)*(EPSY(I)-ZEPY(II,JJ,I))
           TAU(I)=RAN/(1.+PR)*(EPSXY(I)-ZEPXY(II,JJ,I))
           SIGEFF=DSQRT(SIGX(I)*SIGX(I)+SIGY(I)*SIGY(I)+3.*TAU(I)*TAU(I)
     1     -SIGX(I)*SIGY(I))
           RAMP(II,JJ,I)=1.
           IF((SIGEFF-YILSTS) .GT.  O.) GO TO 181
           GO TO 23
 181       CONTINUE
           KH=KH+1
           DEPX=EPSX(I)/(1000.*XKAJ)
           DEPY=EPSY(I)/(1000.*XKAJ)
           DEPXY=EPSXY(I)/(1000.*XKAJ)
 91   SIGX(I)=EO/(1.-NEUO**2.)*(EPSX(I)-EPPX(I)-DEPX+NEUO*(EPSY(I)-
     2 EPPY(I)-DEPY))
      SIGY(I)=EO/(1.-NEUO**2.)*(EPSY(I)-EPPY(I)-DEPY+NEUO*(EPSX(I)-
     2  EPPX(I)-DEPX))
      TAU(I)=2.*GO*(EPSXY(I)-EPPXY(I)-DEPXY)
           AJ=3.
           BJ=DSQRT(AJ)
           DELEPP=2./BJ*DSQRT(DEPX*DEPX+DEPY*DEPY+DEPX*DEPY+DEPXY*DEPXY)
           ALFA=DATAN(ALPHA)
           GAMA=DATAN(EO)
           BETTA=GAMA-ALFA
           YY=DELEPP*DSIN(ALFA)*DSIN(GAMA)/(DSIN(BETTA))
           SIGEFF=YILSTS+YY
 19   CONTINUE
      DEPXN=DELEPP/(2.*SIGEFF)*(2.*SIGX(I)-SIGY(I))
      DEPYN=DELEPP/(2.*SIGEFF)*(2.*SIGY(I)-SIGX(I))
      DEPXYN=(3.*DELEPP)/(2.*SIGEFF)*TAU(I)
           IF(ALPHA .GT. O.) TOL=.0000001
           IF(ALPHA .EQ. O.)   TOL=.00005
           DELEPN=2./BJ*DSQRT(DEPXN*DEPXN+DEPYN*DEPYN+DEPXN*DEPYN+
     1     DEPXYN*DEPXYN)
           IF(DELEPN .GT. 1.) XKAJ=2.*XKAJ
           IF(DELEPN .GT. 1.) GO TO 181
           IF(TOL-DABS(ARD-DELEPN))30,27,27
 30        CONTINUE
           IF(KAR-2 )31,31,222
 222       CONTINUE
           DEPX=(DEPX+DEPXN)/2.
           DEPY=(DEPY+DEPYN)/2.
           DEPXY=(DEPXY+DEPXYN)/2.
           ARD=DELEPN
           GO TO 91
 31        DEPX=DEPXN
      DEPY=DEPYN
      DEPXY=DEPXYN
           ARD=DELEPN
           KAR=KAR+1
        GO TO 91
 27       CONTINUE

           XAM=SIGEFF/(DELEPN+SIGEFF/EO)
           IF(ALPHA .EQ. O.) SF(I)=XAM
           IF(ALPHA.GT. O.)SF(I)=SIGEFF/(YSTAIN+(SIGEFF-YILSTZ)/ALPHA)
           XM=YILSTZ/YSTAIN
           A=SIGEFF/XM
           XBM= DELEPN+SIGEFF/EO
           IF(ALPHA .EQ. O.) B=XBM-A
           IF(ALPHA .GT. O.) B=(YSTAIN+(SIGEFF-YILSTZ)/ALPHA)-A
         F(I)=SIGEFF
           SFF(I)=(1.+2.*PR*A/B)/(2.*(1.+A/B))
             AF(II,JJ,I)=F(I)
             ASF(II,JJ,I)=SF(I)
             ASFF(II,JJ,I)=SFF(I)
           RAMP(II,JJ,I)=O.
           ZEPX(II,JJ,I)=ZEPX(II,JJ,I)+DEPXN
           ZEPY(II,JJ,I)=ZEPY(II,JJ,I)+DEPYN
           ZEPXY(II,JJ,I)=ZEPXY(II,JJ,I)+DEPXYN
  23      CONTINUE

          EPSXY(I)=2.*EPSXY(I)

                    NN=21
       IF(I-NN)25,26,26
  25     I=I+1
          GO TO 24
 26   RETURN
      END
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2

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Doctor of Philosophy

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